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Professor Nikolay Viktorovich Azbelev

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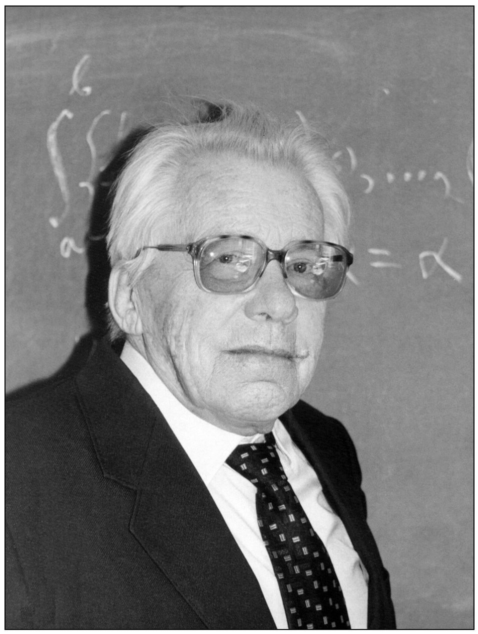

This year we mark the 100th birthday anniversary of the outstanding Russian mathematician Professor Nikolai Viktorovich Azbelev, one of the founders of the national school on integral and differential inequalities, whose works created and successfully developed the modern theory of functional differential equations.

Nikolay Viktorovich was born on April 15, 1922 in the village of Bazlovo, Velikoluksky district, Pskov region in the family of a doctor. His mother, Antonina Fedorovna Khlebnikova, graduated in 1916 from the higher women's courses in St. Petersburg, a student and employee of the famous botanist V. L. Komarov, later the president of the USSR Academy of Sciences. His father, Viktor Nikolaevich Azbelev, graduated in 1905 from Military Medical Academy in St. Petersburg, in 1912 attended courses in microbiology at the Robert Koch Institute in Berlin, was a military doctor in the Far East, then in Samsonov's army during the First World War; Professor, director of the Polar Institute of Bacteriology in Arkhangelsk. During the repressions that followed the murder of S.M. Kirov, he was exiled with his family to Tomsk. Grandfather of Nikolai Viktorovich, Admiral Nikolai Pavlovich Azbelev, was a scientist, teacher of the Naval Academy, founding member of the Russian Astronomical Society, and his great-grandmother, Anna Mikhailovna Zhemchuzhnikova, was the sister of the famous poet A. M. Zhemchuzhnikov.

Nikolai Viktorovich entered the Faculty of Mechanics and Mathematics of Moscow University (MSU) in 1941. From the third year he was drafted into the active army in 1943. After the concussion received in 1944, he returned to the university, and in 1945 he entered the correspondence department of the engine-building faculty of the Moscow Aviation Institute. In 1949, he received a diploma of mechanical engineer at the Aviation Institute. In 1947–1949, he worked as a designer in the design bureau of Academician A. A. Mikulin. In this Design Bureau, Nikolai Viktorovich went through a good school and solved a number of urgent technical problems by mathematical calculations: he proposed an original method for calculating the strength of a radial thrust ball bearing with "multipoint touch", was one of the first to apply in 1947 the method of electrical analogies to the calculation of turbine dynamics and designed a computer for calculating the natural oscillation frequencies of turbojet shafts. In 1951, he entered post-graduate school at the Department of Mathematics of the Moscow Machine Tool Institute. His supervisor was Professor B. I. Segal. In 1954, he defended his PhD thesis at Moscow University, solving the Tchaplygin–Luzin problem on the limits of applicability of the differential inequality theorem. He was sent to Izhevsk to the newly opened Izhevsk Mechanical Institute (IMI), where he headed the Department of Higher Mathematics until 1966.

Almost immediately, after the arrival of Nikolai Viktorovich, an active research team appeared in provincial Izhevsk, called the Izhevsk Mathematical Seminar. The seminar quickly gained credibility among mathematicians, and scientists from other cities began to attend its sessions. The work of the Seminar had an impact on the level of mathematical culture of teaching and research of technical sciences. The short- and long-term courses were organized here to improve the mathematical qualification of engineers, and special meetings were held to discuss the problems related to mathematics that arose in the rapidly developing industry of Izhevsk. Nikolai Viktorovich attached great importance to the training of young mathematicians. In 1962, on the initiative of N. V. Azbelev, the first mathematical class was created at school No. 27 in Izhevsk with the class teacher E. N. Anisimova, a well-known teacher-mathematician. In 1965, the graduates of this class created the first mathematical team at the instrument-making faculty in the specialty 'mathematician – engineer'.

The topic of the Seminar was devoted to the Tchaplygin–Luzin problem related to the Cauchy problem and boundary value problems for ordinary differential and difference equations. Academician N. N. Luzin attached great importance to the problem of limits of applicability of Tchaplygin's theorem, who urged mathematicians to deal with. Articles of the Seminar participants began to appear in scientific journals, including *"Doklady Akademii Nauk SSSR (Soviet Math. Dokl.)"*. Based on the materials of the Seminar, the collection *''Proceedings of the Izhevsk Mathematical Seminar"* began to be published regularly.

In the middle of the XX century, there has been a noticeably increasing interest in the boundary value problems for ordinary differential equations, but there has not yet been a clear definition of the general linear boundary value problem. Nikolai Viktorovich devoted a special course of lectures to the boundary value problems with general boundary conditions, which was very popular among students and post-graduates. Texts of the lectures were distributed between them in a handwritten form. In this course, the concepts appeared that have now become generally accepted: "determinant of the boundary value problem" and "Green's function" for the general case of boundary conditions.

In the linear case, an exhaustive solution was found to the problem of limits of applicability of Tchaplygin's theorem on differential inequality. Applied to a general linear boundary value problem

$$(\mathcal{L}x)(t) = f(t), \quad t \in [a, b], \quad \ell x = 0$$

with boundary conditions defined by the linear vector-functional ℓ, the validity of the differential inequality theorem is guaranteed by the non-negativity of the Green function $G(t,s)$, which leads to the integral representation $x(t) = \int\limits_a^b G(t,s)f(s)\,ds$ of solutions x to this problem. Indeed, if a trial function z satisfies the differential inequality $\mathcal{L}z - f = \varphi \geq 0$, with $\ell z = 0$, then $z(t) - x(t) = \int\limits_a^b G(t,s)\varphi(s)\,ds \geq 0$. Such a solution to the problem required the development of effective methods for studying the sign of the Green function for various classes of boundary conditions and, in the case of multipoint boundary conditions, establishing the connection of this property with the distribution of zeros of solutions to the homogeneous equation.

The boundaries of applicability of the differential inequality theorem sometimes turn out to be too narrow. Thus there arose the problem of constructing a pair of functions z and v on a given interval such that, for a solution x to the equation under consideration, the inequalities $v(t) \leq x(t) \leq z(t)$ hold. This problem of constructing such a pair was called "Tchaplygin's problem". Nikolai Azbelev's doctoral dissertation “On Tchaplygin’s problem”, which he defended in 1962 at Kazan University (with V. V. Nemytsky and L. D. Kudryavtsev as the opponents), is devoted to solving this problem.

In 1966, N. V. Azbelev was elected to the post of Head of the Department of Mathematics of the Tambov Institute of Chemical Engineering (TICE), which he headed until 1975. Nikolai Viktorovich left for Tambov with his wife Lina Fazylovna Rakhmatullina, associate professor of the Department of Higher Mathematics. They were followed by a group of students, and soon a very active seminar appeared in Tambov, dedicated to equations with a deviating argument and boundary value problems. The Tambov Mathematical Seminar continued the traditions of the Izhevsk Seminar.

Thanks to his energy, optimism and wealth of scientific ideas, Nikolai Viktorovich managed to involve in research work not only the staff of the department, but also students, as well as many mathematicians who lived in Tambov.

Soon, a post-graduate course was opened at the Tambov Institute of Chemical Engineering.

Considering that TICE is a specialized engineering university with standard mathematical training of students, Nikolay Viktorovich has achieved the expansion of the mathematical training program at one of the most prestigious faculty, faculty of automation of chemical production processes. After careful selection, a special group of students was formed, whose mathematical training was conducted according to an almost university program. In the future, lectures in special subjects were delivered at a very high level in this group.

Various seminars were organized at the department to attract students to scientific work in the field of mathematics. For first-year students, they were devoted to mathematical analysis and the history of mathematics. Second-year students attended seminars on mathematical physics. Seminars on the control theory were being hold for senior students, where the staff of the departments of Mathematics and Automation participated. Scientific researches carried out at the institute were discussed, the latest results in the field of control obtained by the Soviet and foreign scientists were studied. Many students were transferred to an individual learning plan and attended the “adult” seminar.

With the arrival of N.V. Azbelev, the collection “Works of TICE” began to be published, and annual scientific conferences of teachers and students of TICE gathered mathematicians from many cities of the USSR and actually turned into the all-Union ones.

Nikolay Viktorovich and Lina Fazylovna read lectures at a part of the seminar sessions, while everyone could get a typewritten text. In fact, the material of these lectures consisted of the recent results on boundary value problems and the theory of equations with a deviating argument that was being created. The experience of studying boundary value problems and the analysis of known approaches to the study of equations with a deviating argument led the Tambov seminar to the need to revise the traditional definition of the solution concept.

Already in the simplest case, the equation with a deviating argument

$$\begin{aligned} &\dot{x}(t) = p(t)x[h(t)] + v(t), \quad t \in [a,b], \\ &x(\xi) = \varphi(\xi), \quad \xi \notin [a,b], \end{aligned}$$

where the deviating argument $h(t)$ takes values outside the segment $[a,b]$, requires the “initial function”. The traditional definition of the solution to such an equation contains the requirement of the condition “continuous matching”: $x(a) = \varphi(a)$, $x(b) = \varphi(b)$, which is redundant from the point of view of defining all operations in writing the equation. The new concept is based on the introduction into the description of the equation of the linear operator, “internal superposition” S_h, defined by the equality

$$(S_h y)(t) = \begin{cases} y[h(t)] & \text{if } h(t) \in [a,b], \\ 0 & \text{if } h(t) \notin [a,b]. \end{cases}$$

After introducing the function

$$\varphi^h(t) = \begin{cases} 0 & \text{if } h(t) \in [a,b], \\ \varphi[h(t)] & \text{if } h(t) \notin [a,b], \end{cases}$$

the equation can be written in an equivalent form

$$(\mathcal{L}x)(t) \equiv \dot{x}(t) - p(t)(S_h x)(t) = f(t),$$

where $f(t) = v(t) + p(t)\varphi^h(t)$, and the initial function φ finds its place on the right-hand side of the equation, and the left-hand side of the equation presents a linear operator defined on the set of absolutely continuous on the segment $[a, b]$ functions.

The new concept has opened up the possibility to make full use of the ideas and approaches of functional analysis. In particular, the effective methods of studying the boundary value problems developed by the Tambov Seminar for the differential equation

$$(\mathcal{L}x)(t) \equiv x^{(n)}(t) - \sum_{k=0}^{n-1} p_k(t) x^{(k)}(t) = f(t), \quad t \in [a, b],$$

turned out to be directly applicable to the linear equation

$$(\mathcal{L}x)(t) \equiv \dot{x}(t) - p(t)(S_h x)(t) = f(t)$$

and its natural generalization of the form

$$(\mathcal{L}x)(t) \equiv x^{(n)}(t) - \sum_{k=0}^{n-1} p_k(t)(S_{h_k} x^{(k)})(t) = f(t).$$

The fundamental results obtained in this direction are described in [27, 28, 29]. The rejection of ingrained definitions of the solution concept has revolutionized the doctrine of equations with a deviating argument. By this time, there arose a need for a general approach to numerous classes of equations with respect to differentiable functions such as differential, integro-differential, with a deviating argument and their hybrids. These equations were united by the name "functional differential equations", but did not have a general theory. The next important stage of the seminar was to build the foundations of such a theory. The active work of the Seminar and the creative abilities of Professor N. V. Azbelev allowed the staff of the department to become a recognized scientific school on functional differential equations both in the USSR and abroad.

The activity of the seminar ended with the creation of an effective theory of differential equations with a deviating argument, on the basis of which the modern theory of functional differential equations then arose.

In 1975, Nikolai Viktorovich accepted the invitation of the rector of the Perm Polytechnic Institute (PPI), Professor M. N. Dedyukin, and headed the newly organized Department of Mathematical Analysis. This department was composed mainly of Tambov pupils and collaborators of Nikolai Viktorovich. The research of the Tambov seminar was continued by the Perm Seminar [57], which soon turned into a kind of free (on a voluntary basis) research institute, whose employees were graduate students, doctoral students and trainees of the department. The seminar became the all-Union center for research on functional differential equations, published annual collections of scientific papers *"Boundary Value Problems"* and *"Functional Differential Equations"*, held annual conferences of the all-Union scale. Over 30 years, the Seminar participants defended 12 doctoral and more than 50 candidate dissertations, eight monographs in Russian, among them three in English and one in Portuguese, were published in prestigious publishing houses.

Nikolay Viktorovich has been the permanent head of the Department of Mathematical Analysis at the Perm Polytechnic Institute for 20 years. The participants of the Perm seminar, which has become widely known, under the leadership of N. V. Azbelev, developed a modern theory of functional differential equations (FDE). In 1991, the publishing house "Nauka", Moscow, published a monograph by N. V. Azbelev (together with V. P. Maksimov and L. F. Rakhmatullina) *"Introduction to the theory of functional differential equations"* [1], which at present remains one of the most frequently cited by experts in functional differential equations. The results of further development of the theory of FDE are reflected in nine monographs written by him together with his pupils and collaborators, four of them were published abroad in English.

Today, the theory of functional differential equations

$$\dot{x} = Fx$$

with the operator F, acting from the space $\mathbf{AC}$ of absolutely continuous functions into the space $\mathbf{L}$ of summable functions, covers numerous classes of equations with ordinary derivatives, including integro-differential equations, delayed-argument equations and other classes of equations arising in applications. In the modern theory of FDE, Nikolai Viktorovich owns fundamental results related to various aspects: from the study of general theory issues to the subtle and elegant theorems devoted to special issues (conservation of the sign of the Green function of a linear boundary value problem, oscillatory properties of solutions, signs of stability). The most complete results are obtained for the linear equation

$$(\mathcal{L}x)(t) = f(t)$$

with the operator $\mathcal{L} : \mathbf{AC} \to \mathbf{L}$, the "principal part" [1] of which is a Fredholm operator (i.e., representable in the form of a sum of invertible and completely continuous operators).

Of particular note is the contribution of N. V. Azbelev to the formation and development of the theory of abstract functional differential equation (AFDE), further substantial generalization of equations with ordinary derivatives, covering wide classes of functional differential n-th order equations, systems with impulse perturbations, singular equations. In concrete implementations of the general theory of FDE, such properties of operators in Lebesgue spaces as complete continuity, being Fredholm, integral representations play the certain role.

For some operators arising in modern problems, these properties are not fulfilled in Lebesgue spaces, but they are fulfilled in some other Banach spaces. Such tasks are not included in the framework of the theory of equations $\dot{x} = Fx$ with the operator $F : \mathbf{AC} \to \mathbf{L}$ constructed by virtue of the seminar.

Therefore, it was natural to try to replace the space of absolutely continuous functions with a more general one and replace the Lebesgue space $\mathbf{L}$ with an arbitrary Banach space.

In the work of N. V. Azbelev and L. F. Rakhmatullina [3], there are formulated the foundations of a general theory in which the idea of generalizing the functional differential equation is as follows.

Absolutely continuous function $x : [a, b] \to \mathbf{R}^n$ is a function that allows the decomposition

$$x(t) = \int_a^t \dot{x}(s)\,ds + x(a).$$

Therefore, each pair $\{z, \alpha\} \in \mathbf{L} \times \mathbf{R}^n$ defines the absolutely continuous function

$$x(t) = \int_a^t z(s)\,ds + \alpha,$$

and to each such function x there corresponds the pair $\{z, \alpha\} \in \mathbf{L} \times \mathbf{R}^n$, where $z = \dot{x}$, $\alpha = x(a)$. Thus the space of absolutely continuous functions is isomorphic to the direct product of $\mathbf{L} \times \mathbf{R}^n$. This isomorphism plays an important role in the theory of functional differential equations, allowing, in particular, to reduce many questions about functional differential equation to the relevant questions about the equation in the space $\mathbf{L}$. Replacing the Lebesgue space $\mathbf{L}$ with an arbitrary Banach space $\mathbf{B}$, consider the space $\mathbf{D}$, isomorphic to the direct product $\mathbf{B} \times \mathbf{R}^n$ ($\mathbf{D} \simeq \mathbf{B} \times \mathbf{R}^n$). Such a space turned out to be a far-reaching generalization of the space of absolutely continuous functions, and the equation in this space began to be considered as an "abstract functional differential equation". This theory made it possible to transfer the main statements of the theory about boundary value problems and control problems to new classes of functional differential equations, including equations with impulse effects, singular equations, systems with continuous and discrete time (hybrid systems).

Scientific creativity of N. V. Azbelev is distinguished by non-standard thinking, geometric intuition, simplicity of solving complex problems and dislike for all kinds of speculative near-scientific constructions.

Nikolay Viktorovich has always devoted a lot of effort and attention to mathematical education and training of highly qualified personnel. The first mathematical classes for gifted schoolchildren and special groups of students with enhanced mathematical training in Izhevsk and Tambov were organized thanks to the enthusiasm of Nikolai Viktorovich. The undoubted qualities of a leader constantly created groups of people around Nikolai Viktorovich, united by an interest in a particular problem, scientific or organizational; he always knew how to unite young mathematicians and all those who are passionate about science around him. The Department of Mathematical Analysis of PPI created by him was officially recognized as the basic department for the training of highly qualified personnel for universities of the Ural region, from 1979 to 2004, 30 candidates and 12 doctors of sciences were trained. Since 1994 to his last day, N. V. Azbelev had been the head of the Research Center on Functional Differential Equations at Perm State Technical University (former PPI).

N. V. Azbelev was an active member of the editorial boards of the journals “Differential Equations” (more than 20 years), “Nonlinear Dynamics and System Theory”, “Memoirs on Differential Equations and Mathematical Physics”, “Functional Differential Equations” and many other periodicals.

Scientific and pedagogical activity of N. V. Azbelev has found recognition in Russia and abroad. He was awarded orders and medals, recognized as a Meritorious Science Worker of the Russian Federation, awarded the Grant of the Russian Federation President for Leading Scientist, was a laureate of the State Scientific Scholarship of the Presidium of the Russian Academy of Sciences, was elected an honorary member of the Academy of Nonlinear Sciences, honorary professor of Izhevsk State Technical University, awarded the title “Georg Soros Emeritus Professor”. He has repeatedly acted as an invited lecturer in scientific centers of the world and at major international forums.

Today, the work to which Nikolai Viktorovich devoted his life is being continued by his numerous pupils working in Russia and far beyond its borders, all for whom he was and remains the Teacher.

The works of N.V. Azbelev will inspire for a long time those who continue to study the functional differential equations, find new problem statements and new relevant applications of the theory.

V. P. Maksimov, I. T. Kiguradze, A. V. Ponosov

List of Publications of N. V. Azbelev

(i) Monographs

1. *Introduction to the Theory of Functional-Differential Equations* (with V. P. Maksimov and L. F. Rakhmatullina). (Russian) "Nauka", Moscow, 1991.
2. *Introduction to the Theory of Linear Functional-Differential Equations* (with V. Maksimov and L. Rakhmatullina). Advanced Series in Mathematical Science and Engineering, 3. World Federation Publishers Company, Atlanta, GA, 1995.
3. Theory of linear abstract functional-differential equations and applications (with L. F. Rakhmatullina). *Mem. Differential Equations Math. Phys.* **8** (1996), 102 pp.
4. *Methods of the Contemporary Theory of Linear Functional Differential Equations* (with V. P. Maksimov and L. F. Rakhmatullina). (Russian) Regular and Chaotic Dynamics, Izhevsk, 2000.
5. *Stability of Solutions of Equations with Ordinary Derivatives* (with P. M. Simonov). (Russian) Perm University Publishing, Perm, 2001.
6. *Elements of the Modern Theory of Functional Differential Equations. Methods and Applications* (with V. P. Maksimov and L. F. Rakhmatullina). (Russian) Institute for Computer Studies, Moscow, 2002.
7. *Stability of Differential Equations with Aftereffect* (with P. M. Simonov). Stability and Control: Theory, Methods and Applications, 20. *Taylor & Francis, London*, 2003.
8. *Functional Differential Equations and Variational Problems* (with S. Yu. Kultyshev and V. Z. Tsalyuk). R&C Dynamics, 2006.
9. Introduction to the theory of functional differential equations: methods and applications (with V. P. Maksimov and L. F. Rakhmatullina). Contemporary Mathematics and Its Applications, 3. *Hindawi Publishing Corporation, Cairo*, 2007.

(ii) Selected papers

10. A process of successive approximations for finding characteristic values and characteristic vectors (with R. Vinograd). (Russian) *Doklady Akad. Nauk SSSR (N.S.)* **83** (1952), 173–174.
11. On an approximate solution of ordinary differential equations of the nth order based upon S. A. Chaplygin's method. (Russian) *Doklady Akad. Nauk SSSR (N.S.)* **83** (1952), 517–519.
12. On the limits of applicability of S. A. Chplygin's theorem. (Russian) *Doklady Akad. Nauk SSSR (N.S.)* **89** (1953), 589–591.
13. On a sufficient condition for applicability of Chaplygin's method to equations of higher orders. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **99** (1954), 493–494.
14. On extension of Chaplygin's method beyond the limit of applicability of the theorem on differential inequalities. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **102** (1955), 429–430.
15. On limits of applicability of the theorem of Chaplygin on differential inequalities. (Russian) *Mat. Sb. N.S.* **39(81)** (1956), 161–178.
16. Theorem about the estimation of error of an approximate solution of a differential equation (with L. V. Tonkov). (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **111** (1956), 515–516.
17. On the question of estimating the number of zeros for solutions to the equation $y''' + p(x)y' + q(x)y = 0$. (Russian) *Nauchn. Dokl. Vyssh. Shkoly, Fiz.-Mat. Nauki*, 1958, no. 3, 3–5.
18. Some conditions of the solvability of the Chaplygin problem for the ordinary differential system. (Russian) *Nauchn. Dokl. Vyssh. Shkoly, Fiz.-Mat. Nauki*, 1958, no. 6, 30–35.
19. On the question of the distribution of the zeros of solutions of a third-order linear differential equation (with Z. B. Tsalyuk). (Russian) *Mat. Sb. (N.S.)* **51 (93)** (1960), 475–486.

20. On integral inequalities. I (with Z. B. Tsalyuk). (Russian) *Mat. Sb. (N.S.)* **56 (98)** (1962), 325–342.

21. Theorems on a differential inequality for boundary-value problems (with A. Ya. Khokhryakov and Z. B. Tsalyuk). (Russian) *Mat. Sb. (N.S.)* **59 (101)** (1962), 125–144.

22. Integral and differential inequalities (with Z. B. Tsalyuk). (Russian) 1964 *Proc. Fourth All-Union Math. Congr. (Leningrad, 1961) (Russian), Vol. II*, pp. 384–391 *Izdat. "Nauka", Leningrad.*

23. Necessary and sufficient condition for the boundedness of solutions of a class of systems of linear differential equations (with Z. B. Tsalyuk). (Russian) *Prikl. Mat. Meh.* **28** 149–150; translated in *J. Appl. Math. Mech.* **28** (1964), 173–175.

24. On the problem of a differential inequality (with Z. B. Tsalyuk). (Russian) *Differentsial'nye Uravneniya* **1** (1965), 431–438.

25. On the question of a definition of the concept of a solution of an integral equation with a discontinuous operator (with Li Mun Su and R. K. Ragimhanov). (Russian) *Dokl. Akad. Nauk SSSR* **171** (1966), 247–250; translation in *Sov. Math., Dokl.* **7** (1966), 1437–1440.

26. Integral equations with a discontinuous operator (with R. K. Ragimhanov and L. N. Fadeeva). (Russian) *Differentsial'nye Uravneniya* **5** (1969) 862–873.

27. Integral equations with deviating argument (with M. P. Berdnikova and L. F. Rahmatullina). (Russian) *Dokl. Akad. Nauk SSSR* **192** (1970), 479–482.

28. Linear equations with deviating argument (with L. F. Rahmatullina). (Russian) *Differentsial'nye Uravneniya* **6** (1970), 616–628.

29. The Cauchy problem for differential equations with retarded argument (with L. F. Rahmatullina). (Russian) *Differentsial'nye Uravneniya* **8** (1972), 1542–1552.

30. On the question of differential inequalities for equations with retarded argument (with Ju. I. Zubko and S. M. Labovskiĭ). (Russian) *Differentsial'nye Uravneniya* **9** (1973), 1931–1936.

31. Linear boundary value problems for functional-differential equations. (Russian) *Differentsial'nye Uravneniya* **10** (1974), 579–584.

32. On the question of the stability of the solutions of differential equations with retarded argument (with T. S. Sulavko). (Russian) *Differentsial'nye Uravneniya* **10** (1974), 2091–2100.

33. A certain class of functional-differential equations (with G. G. Islamov). (Russian) *Differentsial'nye Uravneniya* **12** (1976), no. 3, 417–427.

34. Nonlinear functional-differential equations. (Russian) *Differentsial'nye Uravneniya* **12** (1976), no. 11, 1923–1932.

35. A linear functional-differential equation of evolution type (with L. M. Berezanskiĭ and L. F. Rakhmatullina). (Russian) *Differentsial'nye Uravneniya* **13** (1977), no. 11, 1915–1925.

36. A linear functional-differential equation of evolution type. (with L. F. Rahmatullina). (Russian) *Differentsial'nye Uravneniya* **14** (1978), no. 5, 771–797.

37. A priori estimates of the solutions of the Cauchy problem and the solvability of boundary value problems for equations with retarded argument (with V. P. Maksimov). (Russian) *Differentsial'nye Uravneniya* **15** (1979), no. 10, 1731–1747; translation in *Differ. Equations* **15** (1980), 1231–1243.

38. Equations with retarded argument (with V. P. Maksimov). (Russian) *Differentsial'nye Uravneniya* **18** (1982), no. 12, 2027–2050; translation in *Differ. Equations* **18** (1983) , 1419–1441.

39. Some tendencies towards generalizations of a differential equation. (Russian) *Differentsial'nye Uravneniya* **21** (1985), no. 8, 1291–1304; translation in *Differ. Equations* **21** (1985), 871–882.

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41. On the question of linear differential inequalities (with A. I. Domoshnitsky). (Russian) Part I – *Differentsial'nye Uravneniya* **27** (1991), no. 3, 376–384; translation in *Differential Equations* **27** (1991), no. 3, 257–263; Part II – *Differentsial'nye Uravneniya* **27** (1991), no. 6, 923–931; translation in *Differential Equations* **27** (1991), no. 6, 641–647.
42. The stability of linear systems with aftereffect (with L. M. Berezanskii, P. M. Simonov and A. V. Chistyakov). (Russian) Part I – *Differentsial'nye Uravneniya* **23** (1987), no. 5, 745–754; Part II – *Differentsial'nye Uravneniya* **27** (1991), no. 4, 555–562; translation in *Differential Equations* **27** (1991), no. 4, 383–388; Part III – *Differentsial'nye Uravneniya* **27** (1991), no. 10, 1659–1668; translation in *Differential Equations* **27** (1991), no. 10, 1165–1172 (1992); Part IV – *Differentsial'nye Uravneniya* **29** (1993), no. 2, 196–204; translation in *Differential Equations* **29** (1993), no. 2, 153–160.
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