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Short Communication

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ON THE SOLVABILITY OF MULTIPOINT BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF NONLINEAR DIFFERENCE EQUATIONS

Abstract. The effective sufficient conditions are given for the solvability of the multipoint boundary value problems for systems of nonlinear difference equations.

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Let m_0 be a fixed natural number, $\mathbb{N}_{m_0} = \{1, \dots, m_0\}$ and $\widetilde{\mathbb{N}}_{m_0} = \{1, \dots, m_0\}$. Consider the problem of finding a vector-function $y = (y_i)_{i=1}^n : \widetilde{\mathbb{N}}_{m_0} \to \mathbb{R}^n$ satisfying the system of difference equations

$$\Delta y_i(k-1) = g_i\big(k, y_1(k), \dots, y_n(k), y_1(k-1), \dots, y_n(k-1)\big) \quad \text{for } k \in \mathbb{N}_{m_0} \;\; (i = 1, \dots, n) \tag{1}$$

and the multipoint boundary value problem of the Caucy–Nicoletti's type

$$y_i(k_i) = \xi_i(y_1, \dots, y_n) \;\; (i = 1, \dots, n), \tag{2}$$

where $k_i \in \widetilde{\mathbb{N}}_{m_0}$ $(i = 1, \dots, n)$, $g_i(k, \cdot\,) \in C(\mathbb{R}^{2n}, \mathbb{R})$ $(k = 1, \dots, m_0)$, and $\xi_i : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$ $(i = 1, \dots, n)$ are continuous functional, in general nonlinear.

In the paper some effective sufficient conditions are given for the solvability and unique solvability of the boundary value problem (1), (2). Some results of the same type, among them necessary and sufficient condition, are given in [1]. The general nonlinear boundary problems for the difference system (1) is considered in [5], where the Conti–Opial's type existence and uniqueness theorems are given for the problem.

The various question for the linear and nonlinear boundary value problems for the systems of difference equations are considered in [1, 2, 5, 7, 11] (see also the references therein). The questions, analogous to considered in the paper and in [1], are studied sufficiently well (see, for example, [8, 9]) for the boundary value problems for the ordinary differential systems, and in [3, 4, 6, 10, 12] for the impulsive systems (see also the references therein).

We realize the results for the boundary condition

$$y_i(t_i) = c_i \quad (i = 1, \dots, n), \tag{3}$$

where $c_i \in \mathbb{R}^n$ $(i = 1, \dots, n)$ are constant vectors.

Throughout the paper the following notation and definitions will be used.

$\mathbb{R} =]-\infty, +\infty[$, $\mathbb{R}_+ = [0, +\infty[$; $[a, b]$ $(a, b \in \mathbb{R})$ is a closed interval.

$\mathbb{R}^{n\times m}$ is the space of all real $n \times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm $\|X\| = \max\limits_{j=1,\dots,m} \sum\limits_{i=1}^{n} |x_{ij}|$.

$\mathbb{R}_+^{n\times m} = \big\{(x_{ij})_{i,j=1}^{n,m} : \ x_{ij} \geq 0 \ (i = 1, \dots, n; \ j = 1, \dots, m)\big\}$.

$\mathbb{R}^n = \mathbb{R}^{n\times 1}$ is the space of all real column n-vectors $x = (x_i)_{i=1}^n$; $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$.

If $l \in \mathbb{N}$, then $\mathbb{N}_l = \{1, \dots, l\}$, $\widetilde{\mathbb{N}}_l = \{0, 1, \dots, l\}$.

$E(J, \mathbb{R}^{n\times m})$, where $J \subset \mathbb{Z}$, is the space of all matrix-functions $Y = (y_{ij})_{i,j=1}^{n,m} : J \to \mathbb{R}^{n\times m}$ with the norm $\|Y\|_J = \max\{\|Y(k)\| : \ k \in J\}$, $\|Y\|_{\nu,J} = \big(\sum\limits_{k\in J} \|Y(k)\|^\nu\big)^{\frac{1}{\nu}}$ if $1 \leq \nu < +\infty$, and$\|Y\|_{+\infty,\alpha} = \|Y\|_J$.

Δ is the difference operator of the first order, i.e.

$$\Delta Y(k-1) = Y(k) - Y(k-1) \quad \text{for } \ Y \in E(\widetilde{\mathbb{N}}_l, \mathbb{R}^{n\times m}), \ \ k \in \mathbb{N}_l.$$

If a function Y is defined on $\mathbb{N}_l$ or $\widetilde{\mathbb{N}}_{l-1}$, then we assume $Y(0) = O_{n\times m}$, or $Y(l) = O_{n\times m}$, respectively, if it is necessary.

If B_1 and B_2 are normed spaces, then an operator $\xi : B_1 \to B_2$ (nonlinear, in general) is called positive homogeneous if $\xi(\lambda x) = \lambda\xi(x)$ for every $\lambda \in \mathbb{R}_+$ and $x \in B_1$. If the spaces B_1 and B_2 are partial ordered then the operator ξ is called nondecreasing if the inequality $\xi(x) \leq \xi(y)$ holds for every $x, y \in B_1$ such that $x \leq y$.

A matrix-function is said to be continuous, nondecreasing, integrable, etc., if each of its components is such.

Definition 1. Let $k_1, \dots, k_n \in \widetilde{\mathbb{N}}_{m_0}$. We say that the triplet $(Q_1, Q_2; \xi_0)$, consisting of matrix-functions $Q_j = (q_{jil})_{i,l=1}^n \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$ and a positive homogeneous nondecreasing continuous vector-functional $\xi_0 = (\xi_{0i})_{i=1}^n : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}_+^n) \to \mathbb{R}_+^n$, belongs to the set $U(k_1, \dots, k_n)$ if $q_{jil}(t) \geq 0$ $(j = 1, 2; \ i \neq l; \ i, l = 1, \dots, n)$ and the system of difference inequalities

$$\Delta y_i(k-1) \operatorname{sgn}\Big(k - k_i - \frac{1}{2}\Big) \leq \sum_{l=1}^{n} \big(q_{1il} y_l(k) + q_{2il} y_l(k-1)\big) \ \ (i = 1, \dots, n),$$

has no nontrivial nonnegative solution satisfying the condition

$$y_i(k_i) \le \xi_{0i}(|y_1|, \dots, |y_n|) \quad (i = 1, \dots, n).$$

The set analogous to $U(k_1, \dots, k_{m_0})$ has been introduced by I. Kiguradze for ordinary differential equations (see [8, 9]).

Theorem 1. *Let the inequalities*

$$\begin{aligned} g_i(t, y_1, \dots, y_{2n}) \operatorname{sgn}\Big[\Big(k - k_i - \frac{1}{2}\Big) y_{j\,n+i}\Big] &\\ \le \sum_{l=1}^{n} \big(p_{1il}(k)|y_l| + p_{2il}(k)|y_{n+l}|\big)& \\ + q_i\Big(k, \sum_{l=1}^{2n} |y_l|\Big) \quad (j = 0, 1;\ \ k = 1, \dots, m_0;\ \ i = 1, \dots, n)& \end{aligned} \tag{4}$$

and

$$|\xi_i(y_1, \dots, y_n)| \le \xi_{0i}\big(|y_1|, \dots, |y_n|\big) + \gamma_i\Big(\sum_{l=1}^{n} |y_l|\Big) \quad (i = 1, \dots, n)$$

be fulfilled on the sets $\mathbb{R}^{2n}$ and $E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$, respectively, where $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, and $q_i(k, \cdot\,) \in C(\mathbb{R}_+, \mathbb{R}_+)$ and $\gamma_i \in C(\mathbb{R}_+, \mathbb{R}_+)$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$ are nondecreasing functions satisfying the conditions

$$\lim_{\rho\to+\infty} \frac{q_i(k, \rho)}{\rho} = \lim_{\rho\to+\infty} \frac{\gamma_i(\rho)}{\rho} = 0 \ \ \text{for}\ \ k \in \mathbb{N}_{m_0} \quad (i = 1, \dots, n), \tag{5}$$

$\xi_{0i} : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}_+) \to \mathbb{R}_+$ $(i = 1, \dots, n)$ are positive homogeneous nondecreasing functionals. Moreover, let there exist a matrix-functions $Q_j = (q_{jil})_{i,l=1}^n \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$ such that

$$(Q_1, Q_2; \xi) \in U(k_1, \dots, k_{m_0}), \tag{6}$$

here $\xi = (\xi_{0i})_{i=1}^n$, and

$$p_{jil}(k) \le q_{jil}(k) \ \ \text{for}\ \ k \in \mathbb{N}_{m_0} \quad (j = 1, 2; i, l = 1, \dots, n). \tag{7}$$

Then the problem (1), (2) *is solvable.*

Corollary 1. *Let the inequalities* (4) *and*

$$|\xi_i(y_1, \dots, y_n)| \le \sum_{m=1}^{n} l_{im} \|y_m\|_{\nu, \mathbb{N}_{m_0}} + \gamma_i\Big(\sum_{l=1}^{n} |y_l|\Big) \quad (i = 1, \dots, n)$$

be fulfilled on the sets $\mathbb{R}^{2n}$ and $E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$, respectively, where $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, and $q_i(k, \cdot\,) \in C(\mathbb{R}_+, \mathbb{R}_+)$ and $\gamma_i \in C(\mathbb{R}_+, \mathbb{R}_+)$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$ are nondecreasing functions satisfying the condition (5), *$l_{im} \in \mathbb{R}_+$ $(i, m = 1, \dots, n)$, $2 \le \nu \le +\infty$. Moreover, let* *the module of every characteristic value of the matrices* $\mathcal{H} = (h_{jim})_{i,j=1}^n$ $(j = 1, 2)$ *be less then* 1, *where*

$$h_{jim} = (2-j)m_0^{\frac{1}{\nu}} l_{im} + \Big(\frac{1}{2}\sin^{-1}\frac{\pi}{4m_0+2}\Big)^{\frac{2}{\nu}} \|q_{jim}\|_{\mu,\mathbb{N}_{m_0}}$$
$$(j = 1, 2; \;\; i, m = 1, \dots, n),$$

where $\frac{1}{\mu} + \frac{2}{\nu} = 1$. *Then the problem* (1), (2) *is solvable.*

Corollary 2. *Let the inequalities*

$$g_i(t, y_1, \dots, y_{2n}) \operatorname{sgn}\Big[\Big(k - k_i - \frac{1}{2}\Big) y_{j\,n+i}\Big] \le \sum_{l=1}^n \Big(\eta_{1il}(k)|y_l| + \eta_{2il}(k)|y_{n+l}|\Big) + q_i\Big(k, \sum_{l=1}^{2n} |y_l|\Big) \;\; (j = 0, 1; \;\; k = 1, \dots, m_0; \;\; i = 1, \dots, n) \tag{8}$$

and

$$\big|\xi_i(y_1, \dots, y_n)\big| \le \mu_i |y_i(l_i)| + \gamma_i\Big(\sum_{l=1}^n |y_i|\Big) \;\; (i = 1, \dots, n)$$

be fulfilled on the sets $\mathbb{R}^{2n}$ *and* $E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$, *respectively, where* $\eta_{jil} \in \mathbb{R}_+$ $(j = 1, 2;\ i \ne l;\ i, l = 1, \dots, n)$, $-1 < \eta_{jii} < 0$ $(j = 1, 2;\ i = 1, \dots, n)$, $q_i(k, \cdot\,) \in C(\mathbb{R}_+, \mathbb{R}_+)$ *and* $\gamma_i \in C(\mathbb{R}_+, \mathbb{R}_+)$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$ *are nondecreasing functions satisfying the condition* (5), *and* $\mu_i \in \mathbb{R}_+$ *and* $l_i \in \{1, \dots, m_0\}$ $l_i \ne k_i$ $(i = 1, \dots, n)$. *Moreover, let*

$$\mu_i \max\big\{\gamma_{1i}(l_i), \gamma_{2i}(l_i)\big\} < 1 \;\; (i = 1, \dots, n), \tag{9}$$

and the real part of every characteristic value of the matrix $(\xi_{il})_{i,l=1}^n$ *be negative, where*

$$\gamma_{ji}(k) \equiv \big(1 + (-1)^j \eta_{jii} \operatorname{sgn}(k - k_i)\big)^{(-1)^j (k-k_i)} \;\; (j = 1, 2; i = 1, \dots, n),$$
$$\xi_{ii} = \eta_{1ii} + \eta_{2ii}), \;\; \xi_{il} = \eta_{1il} h_{1i} + \eta_{2i} h_{2il} \;\; (i \ne l; \;\; i, l = 1, \dots, n),$$

and

$$h_{1i} = h_{2i} = 1 \;\; \text{if} \;\; 0 \le \mu_i \le 1,$$
$$h_{ji} = 1 + (\mu_i - 1)\big(1 - \mu_i \gamma_{ji}(l_i)\big)^{-1} \;\; \text{if} \;\; \mu_i > 1 \;\; (i = 1, \dots, n).$$

Then the problem (1), (2) *is solvable.*

Theorem 2. *Let the inequalities*

$$\big[g_i(t,y_1,\dots,y_{2n})-g_i(t,z_1,\dots,z_{2n})\big]\operatorname{sgn}\Big[\Big(k-k_i-\frac{1}{2}\Big)(y_{j\,n+i}-z_{j\,n+i})\Big]$$
$$\leq \sum_{l=1}^{n}\Big(p_{1il}(k)|y_l-z_l|+p_{2il}(k)|y_{n+l}-z_{n+l}|\Big) \tag{10}$$
$$(j=0,1;\ \ k=1,\dots,m_0;\ \ i=1,\dots,n)$$

and

$$\big|\xi_i(y_1,\dots,y_n)-\xi_i(z_1,\dots,z_n)\big|$$
$$\leq \xi_{0i}\big(|y_1-z_l|,\dots,|y_n-z_n|\big)\ \ (i=1,\dots,n) \tag{11}$$

be fulfilled on the sets $\mathbb{R}^{2n}$ *and* $E(\widetilde{\mathbb{N}}_{m_0},\mathbb{R}^n)$*, respectively, where* $(p_{jil})_{i,l=1}^n\in E(\mathbb{N}_{m_0},\mathbb{R}^{n\times n})$ $(j=1,2)$*. Moreover, let there exists a matrix-functions* $Q_j=(q_{jil})_{i,l=1}^n\in E(\widetilde{\mathbb{N}}_{m_0},\mathbb{R}^{n\times n})$ $(j=1,2)$ *such that the conditions* (6) *and* (7) *hold, where* $\xi=(\xi_{0i})_{i=1}^n$*. Then the problem* (1), (2) *has one and only one solution.*

Corollary 3. *Let the inequalities* (10) *and*

$$\big|\xi_i(y_1,\dots,y_n)-\xi_i(z_1,\dots,z_n)\big|\leq\sum_{m=1}^{n}l_{im}\|y_m-z_m\|_{\nu,\mathbb{N}_{m_0}}\ \ (i=1,\dots,n)$$

be fulfilled on the sets $\mathbb{R}^{2n}$ *and* $E(\widetilde{\mathbb{N}}_{m_0},\mathbb{R}^n)$*, respectively, where* $(p_{jil})_{i,l=1}^n\in E(\mathbb{N}_{m_0},\mathbb{R}^{n\times n})$ $(j=1,2)$*, and* $l_{im}\in\mathbb{R}_+$ $(i,m=1,\dots,n)$*,* $2\leq\nu\leq+\infty$*. Moreover, let the module of every characteristic value of the matrices* $\mathcal{H}=(h_{jim})_{i,j=1}^n$ $(j=1,2)$*, appearing in the Corollary* 1*, be less then* 1*, where where* $\frac{1}{\mu}+\frac{2}{\nu}=1$*. Then the problem* (1), (2) *has one and only one solution.*

Corollary 4. *Let the inequalities*

$$\big[g_i(t,y_1,\dots,y_{2n})-g_i(t,z_1,\dots,z_{2n})\big]\operatorname{sgn}\Big[\Big(k-k_i-\frac{1}{2}\Big)(y_{j\,n+i}-z_{j\,n+i})\Big]$$
$$\leq\sum_{l=1}^{n}\Big(\eta_{1il}|y_l-z_l|+\eta_{2il}|y_{n+l}-z_{n+l}|\Big)$$
$$(j=0,1;\ \ k=1,\dots,m_0;\ \ i=1,\dots,n)$$

be fulfilled on the sets $\mathbb{R}^{2n}$*, where* $\eta_{jil}\in\mathbb{R}_+$ $(j=1,2;\ i\neq l;\ i,l=1,\dots,n)$*,* $-1<\eta_{jii}<0$ $(j=1,2;\ i=1,\dots,n)$*. Moreover, let* $\mu_i\in\mathbb{R}_+$ *and* $l_i\in\{1,\dots,m_0\}$*,* $l_i\neq k_i$ $(i=1,\dots,n)$*, be such the condition* (9) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where*

$$\gamma_{ji}(k)\equiv\big(1+(-1)^j\eta_{jii}\operatorname{sgn}(k-k_i)\big)^{(-1)^j(k-k_i)}\ \ (j=1,2;i=1,\dots,n),$$
$$\xi_{ii}=\eta_{1ii}+\eta_{2ii},\quad \xi_{il}=\eta_{1il}h_{1il}+\eta_{2il}h_{2il}\ \ (i\neq l;\ \ i,l=1,\dots,n),$$

and

$$h_{1i} = h_{2i} = 1 \;\; \text{if} \;\; 0 \le \mu_i \le 1,$$
$$h_{ji} = 1 + (\mu_i - 1)\big(1 - \mu_i \gamma_{ji}(l_i)\big)^{-1} \;\; \text{if} \;\; \mu_i > 1 \;\; (i = 1, \dots, n),$$

Then the system (1) *has one and only one solution under the condition*

$$y_i(k_i) = \lambda_i y_i(l_i) + \beta_i \;\; (i = 1, \dots, n)$$

for every $\lambda_i \in [-\mu_i, \mu_i]$ *and* $\beta_i \in \mathbb{R}$ $(i = 1, \dots, n)$.

Theorem 3. *Let the matrix functions* $Q_j = (q_{jil})_{i,l=1}^n \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$ *and the linear continuous vector-functional* $\xi_0 = (\xi_{0i})_{i=1}^n : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}_+^n) \to \mathbb{R}_+^n$ *be such that* $q_{jil}(t) \ge 0$ $(j = 1, 2;\ i \ne l;\ i, l = 1, \dots, n)$ *but the condition* (6) *be violated. Then there exist matrix-functions* $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, *functions* $g_i(k, \cdot) \in C(\mathbb{R}^{2n}, \mathbb{R})$ $(k = 1, \dots, m_0)$, *and continuous functionals* $\xi_i : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$ $(i = 1, \dots, n)$ *such that the condition* (7) *hold, the inequalities* (10) *and* (11) *are fulfilled on the sets* $\mathbb{R}^{2n}$ *and* $E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$, *respectively, but the problem* (1), (2) *is not solvable.*

The conditions for the solvability of the problem (1), (3) follows from the theorems and corollaries given above if we assume $\xi_i(y_1, \dots, y_n) \equiv c_i$ $(i = 1, \dots, n)$.

We have the following results for the solvability of the problem (1), (3).

Theorem 4. *Let the inequalities* (4) *be fulfilled on the set* $\mathbb{R}^{2n}$, *where* $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, *and* $q_i(k, \cdot) \in C(\mathbb{R}_+, \mathbb{R}_+)$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$ *are nondecreasing functions satisfying the condition* (5). *Moreover, let there exist a matrix-functions* $Q_j = (q_{jil})_{i,l=1}^n \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$ *such that the condition* (7) *hold, and the system of difference inequalities appearing in the Definition* 1 *has no nontrivial nonnegative solution satisfying the condition*

$$y_i(k_i) = 0 \;\; (i = 1, \dots, n).$$

Then the problem (1), (3) *is solvable.*

Corollary 5. *Let the inequalities* (4) *be fulfilled on the set* $\mathbb{R}^{2n}$, *respectively, where* $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, *and* $q_i(k, \cdot) \in C(\mathbb{R}_+, \mathbb{R}_+)$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$ *are nondecreasing functions satisfying the condition* (5). *Moreover, let the module of every characteristic value of the matrices* $\mathcal{H}_j = (h_{jim})_{i,j=1}^n$ $(j = 1, 2)$ *be less then* 1, *where*

$$h_{jim} = \Big(\frac{1}{2} \sin^{-1} \frac{\pi}{4m_0 + 2}\Big)^{\frac{2}{\nu}} \|q_{jim}\|_{\mathbb{N}_{m_0}} \;\; (j = 1, 2; i, m = 1, \dots, n).$$

Then the problem (1), (3) *is solvable.*

Corollary 6. *Let the inequalities* (8) *be fulfilled on the set* $\mathbb{R}^{2n}$*, where* $\eta_{jil} \in \mathbb{R}_+$ $(j = 1,2;\ i \neq l;\ i,l = 1,\dots,n)$, $-1 < \eta_{jii} < 0$ $(j = 1,2;\ i = 1,\dots,n)$, $q_i(k,\cdot) \in C(\mathbb{R}_+,\mathbb{R}_+)$ $(i = 1,\dots,n;\ k = 1,\dots,m_0)$ *are nondecreasing functions satisfying the condition* (5)*. Moreover, let the real part of every characteristic value of the matrix* $(\eta_{1il} + \eta_{2il})_{i,l=1}^n$ *be negative. Then the problem* (1)*,* (3) *is solvable.*

Theorem 5. *Let the inequalities* (9) *be fulfilled on the set* $\mathbb{R}^{2n}$*, where* $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0},\mathbb{R}^{n\times n})$ $(j = 1,2)$*. Moreover, let there exists a matrix-functions* $Q_j = (q_{jil})_{i,l=1}^n \in E(\widetilde{\mathbb{N}}_{m_0},\mathbb{R}^{n\times n})$ $(j = 1,2)$ *such that the condition* (7) *hold, and the system of difference inequalities, appearing in the Definition* 1*, has no nontrivial nonnegative solution satisfying the condition* (13)*, where* $\xi = (\xi_{0i})_{i=1}^n$*. Then the problem* (1)*,* (3) *has one and only one solution.*

Corollary 7. *Let the inequalities* (10) *be fulfilled on the set* $\mathbb{R}^{2n}$*, where* $(p_{jil})_{i,l=1}^n \in E(\mathbb{N}_{m_0},\mathbb{R}^{n\times n})$ $(j = 1,2)$*. Moreover, let the module of every characteristic value of the matrices* $\mathcal{H} = (h_{jim})_{i,j=1}^n$ $(j = 1,2)$*, appearing in the Corollary* 5*, be less then* 1*. Then the problem* (1)*,* (3) *has one and only one solution.*

Corollary 8. *Let the inequalities* (8) *be fulfilled on the set* $\mathbb{R}^{2n}$*, where* $\eta_{jil} \in \mathbb{R}_+$ $(j = 1,2;\ i \neq l;\ i,l = 1,\dots,n)$, $-1 < \eta_{jii} < 0$ $(j = 1,2;\ i = 1,\dots,n)$*. Moreover, let the real part of every characteristic value of the matrix* $(\eta_{1il} + \eta_{2il})_{i,l=1}^n$ *be negative. Then the problem* (1)*,* (3) *has one and only one solution.*

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