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## Short Communications

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# ON SOLVABILITY OF BOUNDARY VALUE PROBLEMS ON AN INFINITY INTERVAL FOR NONLINEAR TWO DIMENSIONAL GENERALIZED AND IMPULSIVE DIFFERENTIAL SYSTEMS

**Abstract.** Sufficient conditions are given for the solvability of boundary value problems on an infinite interval for nonlinear two dimensional generalized and impulsive differential systems.

[illegible]

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Let $c \in \mathbb{R}$, $a_{ik} : \mathbb{R}_+ \to \mathbb{R}$ $(i,k = 1,2)$ be nondecreasing continuous from the left functions, and let $f_k : \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R}$ be a vector-function belonging to the Carathéodory class corresponding to the $a_{ik}$ for every $i,k \in \{1,2\}$.

In this paper we investigate the question of existence of solutions for the two dimensional generalized differential system

$$dx_i(t) = f_1(t, x_1(t), x_2(t)) \cdot da_{i1}(t) + \\ + f_2(t, x_1(t), x_2(t)) \cdot da_{i2}(t) \quad \text{for} \quad t \in \mathbb{R}_+ \quad (i = 1,2), \tag{1}$$

satisfying one of the following two conditions

$$x_1(0) = c, \quad \sup\Big\{|x_1(t)| + |x_2(t)| : \ t \in \mathbb{R}_+\Big\} < \infty \tag{2}$$

and

$$\sup\Big\{|x_1(t)| + |x_2(t)| : \ t \in \mathbb{R}_+\Big\} < \infty. \tag{3}$$

We give sufficient conditions for the existence of solutions of the boundary value problems (1), (2) and (1), (3). Analogous results are contained in [10], [11], [13]–[17] for ordinary differential and functional differential systems.

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The theory of generalized ordinary differential equations enables one to investigate ordinary differential, impulsive and difference equations from a common point of view (see, e.g., [1]–[9], [12], [23], and references therein).

We realize the obtained result for the following second order system of impulsive equations

$$\frac{dx_i}{dt} = f_i(t, x_1, x_2) \text{ for almost all } t \in \mathbb{R}_+ \setminus \{\tau_1, \tau_2, \dots\} \ (i = 1, 2), \qquad (4)$$

$$x_i(\tau_k+) - x_i(\tau_k-) = \alpha_{ki} I_{ki}(x_1(\tau_k-), x_2(\tau_k-)) \text{ for } k \in \{1, 2, \dots\} \ (i = 1, 2), \quad (5)$$

where $0 < \tau_1 < \tau_2 < \dots$, $\tau_k \to \infty$ $(k \to \infty)$ (we will assume $\tau_0 = 0$ if necessary), $\alpha_{ki} \in \mathbb{R}$ $(i = 1, 2;\ k = 1, 2, \dots)$, $f_i \in K_{loc}(\mathbb{R}_+ \times \mathbb{R}^2, \mathbb{R})$ $(i = 1, 2)$, and $I_{ki} : \mathbb{R}^2 \to \mathbb{R}$ $(i = 1, 2;\ k = 1, 2, \dots)$ are continuous operators.

Throughout the paper the following notation and definitions will be used.

$\mathbb{R} = ]-\infty, +\infty[$, $\mathbb{R}_+ = [0, +\infty[$; $[a, b]$ $(a, b \in \mathbb{R})$ is a closed segment.

$\mathbb{R}^{n\times m}$ is the set of all real $n \times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$.

$\mathbb{R}^n = \mathbb{R}^{n\times 1}$ is the set of all real column $n$-vectors $x = (x_i)_{i=1}^n$, $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$.

$\mathrm{diag}(\lambda_1, \dots, \lambda_n)$ is the diagonal matrix with the diagonal elements $\lambda_1, \dots, \lambda_n$.

$\bigvee\limits_a^b(X)$ is the total variation of the matrix-function $X : [a, b] \to \mathbb{R}^{n\times m}$, i.e., the sum of total variations of the latter's components.

$X(t-)$ and $X(t+)$ are the left and the right limits of the matrix-function $X : [a, b] \to \mathbb{R}^{n\times m}$ at the point $t$ (we will assume $X(t) = X(a)$ for $t \le a$ and $X(t) = X(b)$ for $t \ge b$, if necessary);

$$d_1X(t) = X(t) - X(t-), \quad d_2X(t) = X(t+) - X(t).$$

$\mathrm{BV}([a, b], \mathbb{R}^{n\times m})$ is the set of all matrix-functions of bounded variation $X : [a, b] \to \mathbb{R}^{n\times m}$ (i.e., such that $\bigvee\limits_a^b(X) < +\infty$).

$\mathrm{BV}_{loc}(\mathbb{R}, \mathbb{R}^{n\times m})$ is the set of all matrix-functions $X : \mathbb{R} \to \mathbb{R}^{n\times m}$ for which $\bigvee\limits_a^b(X) < +\infty$) for every $a, b \in \mathbb{R}$ $(a < b)$.

$s_j : \mathrm{BV}([a, b], \mathbb{R}) \to \mathrm{BV}([a, b], \mathbb{R})$ $(j = 0, 1, 2)$ are the operators defined, respectively, by

$$s_1(x)(a) = s_2(x)(a) = 0,$$

$$s_1(x)(t) = \sum_{a<\tau\le t} d_1x(\tau) \text{ and } s_2(x)(t) = \sum_{a\le\tau<t} d_2x(\tau) \text{ for } a < t \le b$$

and

$$s_0(x)(t) = x(t) - s_1(x)(t) - s_2(x)(t) \text{ for } t \in [a, b].$$

$\mathcal{A} : \mathrm{BV}_{loc}(\mathbb{R}, \mathbb{R}) \times \mathrm{BV}_{loc}(\mathbb{R}, \mathbb{R}) \to \mathrm{BV}_{loc}(\mathbb{R}, \mathbb{R})$ is the operator defined by

$$\mathcal{A}(x, y)(0) = 0,$$

$$\mathcal{A}(x,y)(t) = y(t) + \sum_{0<\tau\leq t} d_1x(\tau)\cdot\big(1-d_1x(\tau)\big)^{-1}d_1y(\tau)-$$
$$-\sum_{0\leq\tau<t} d_2x(\tau)\cdot\big(1+d_2x(\tau)\big)^{-1}d_2y(\tau) \ \text{ for } \ t>0,$$
$$\mathcal{A}(x,y)(t) = y(t) - \sum_{t<\tau\leq 0} d_1x(\tau)\cdot\big(1-d_1x(\tau)\big)^{-1}d_1y(\tau)+$$
$$+\sum_{t\leq\tau<0} d_2x(\tau)\cdot\big(1+d_2x(\tau)\big)^{-1}d_2y(\tau) \ \text{ for } \ t<0$$

for every $x \in \mathrm{BV}_{loc}(\mathbb{R},\mathbb{R})$ such that for every $x \in \mathrm{BV}_{loc}(\mathbb{R},\mathbb{R})$ such that

$$1+(-1)^j d_jx(t)\neq 0 \ \text{ for } \ t\in\mathbb{R} \ \ (j=1,2).$$

If $g:[a,b]\to\mathbb{R}$ is a nondecreasing function, $x:[a,b]\to\mathbb{R}$ and $a\leq s<t\leq b$, then

$$\int_s^t x(\tau)\,dg(\tau) = \int_{]s,t[} x(\tau)\,ds_0(g)(\tau) + \sum_{s<\tau\leq t} x(\tau)d_1g(\tau) + \sum_{s\leq\tau<t} x(\tau)d_2g(\tau),$$

where $\int_{]s,t[} x(\tau)\,ds_0(g)(\tau)$ is the Lebesgue–Stieltjes integral over the open interval $]s,t[$ with respect to the measure $\mu_0(s_0(g))$ corresponding to the function $s_0(g)$.

If $a=b$, then we assume

$$\int_a^b x(t)\,dg(t) = 0.$$

If $g(t)\equiv g_1(t)-g_2(t)$, where $g_1$ and $g_2$ are nondecreasing functions, then

$$\int_s^t x(\tau)\,dg(\tau) = \int_s^t x(\tau)\,dg_1(\tau) - \int_s^t x(\tau)\,dg_2(\tau) \ \text{ for } \ s\leq t.$$

$L([a,b],\mathbb{R};g)$ is the set of all functions $x:[a,b]\to\mathbb{R}$ measurable and integrable with respect to the measures $\mu(g_i)$ $(i=1,2)$, i.e., such that

$$\int_a^b |x(t)|\,dg_i(t) < +\infty \ \ (i=1,2).$$

A matrix-function is said to be continuous, nondecreasing, integrable, etc., if each of its components is such.

If $G=(g_{ik})_{i,k=1}^{l,n}:[a,b]\to\mathbb{R}^{l\times n}$ is a nondecreasing matrix-function and $D\subset\mathbb{R}^{n\times m}$, then $L([a,b],D;G)$ is the set of all matrix-functions $X=(x_{kj})_{k,j=1}^{n,m}:[a,b]\to D$ such that $x_{kj}\in L([a,b],\mathbb{R};g_{ik})$ $(i=1,\dots,l;\ k=$

$1,\dots,n;\ j=1,\dots,m)$;

$$\int\limits_s^t dG(\tau)\cdot X(\tau)=\bigg(\sum_{k=1}^n\int\limits_s^t x_{kj}(\tau)dg_{ik}(\tau)\bigg)_{i,j=1}^{l,m} \ \text{ for }\ a\le s\le t\le b,$$

$$S_j(G)(t)\equiv\big(s_j(g_{ik})(t)\big)_{i,k=1}^{l,n}\ \ (j=0,1,2).$$

If $D_1\subset\mathbb{R}^n$ and $D_2\subset\mathbb{R}^{n\times m}$, then $K([a,b]\times D_1,D_2;G)$ is the Carathéodory class, i.e., the set of all mappings $F=(f_{kj})_{k,j=1}^{n,m}:[a,b]\times D_1\to D_2$ such that for each $i\in\{1,\dots,l\}$, $j\in\{1,\dots,m\}$ and $k\in\{1,\dots,n\}$: a) the function $f_{kj}(\cdot,x):[a,b]\to D_2$ is $\mu(g_{ik})$-measurable for every $x\in D_1$; b) the function $f_{kj}(t,\cdot):D_1\to D_2$ is continuous for $\mu(g_{ik})$-almost every $t\in[a,b]$, and $\sup\big\{|f_{kj}(\cdot,x)|:\ x\in D_0\big\}\in L([a,b],\mathbb{R};g_{ik})$ for every compact $D_0\subset D_1$.

If $G_j:[a,b]\to\mathbb{R}^{l\times n}$ $(j=1,2)$ are nondecreasing matrix-functions, $G=G_1-G_2$ and $X:[a,b]\to\mathbb{R}^{n\times m}$, then

$$\int\limits_s^t dG(\tau)\cdot X(\tau)=\int\limits_s^t dG_1(\tau)\cdot X(\tau)-\int\limits_s^t dG_2(\tau)\cdot X(\tau)\ \text{ for }\ s\le t,$$

$$S_k(G)=S_k(G_1)-S_k(G_2)\ \ (k=0,1,2),$$

$$L([a,b],D;G)=\bigcap_{j=1}^2 L([a,b],D;G_j),$$

$$K([a,b]\times D_1,D_2;G)=\bigcap_{j=1}^2 K([a,b]\times D_1,D_2;G_j).$$

$L_{loc}(\mathbb{R},D;G)$ is the set of all matrix-functions $X:\mathbb{R}\to D$ such that its restriction on $[a,b]$ belongs to $L([a,b],D;G)$ for every $a$ and $b$ from $\mathbb{R}$ $(a<b)$.

$K([a,b]\times D_1,D_2;G)$ is the set of all matrix-functions $F=(f_{kj})_{k,j=1}^{n,m}:\mathbb{R}\times D_1\to D_2$ such that its restriction on $[a,b]$ belongs to $K([a,b],D;G)$ for every $a$ and $b$ from $\mathbb{R}$ $(a<b)$.

If $G(t)\equiv\operatorname{diag}(t,\dots,t)$, then we omit $G$ in the notation containing $G$.

The inequalities between the vectors and between the matrices are understood componentwise.

A vector-function $x=(x_i)_1^2\in\mathrm{BV}_{loc}(\mathbb{R}_+,\mathbb{R}^2)$ is said to be a solution of the system (1) if

$$x_i(t)=x_i(s)+\int\limits_s^t f_1(\tau,x_1(\tau),x_2(\tau))\cdot da_{i1}(\tau)+$$

$$+\int\limits_s^t f_2(\tau,x_1(\tau),x_2(\tau))\cdot da_{i2}(t)\ \text{ for }\ 0\le s\le t\ \ (s,t\in\mathbb{R})\ \ (i=1,2).$$

If $s \in \mathbb{R}$ and $\beta \in \mathrm{BV}_{loc}(\mathbb{R},\mathbb{R})$ are such that

$$1 + (-1)^j d_j\beta(t) \neq 0 \ \text{ for } \ (-1)^j(t-s) < 0 \ \ (j=1,2),$$

then by $\gamma_\beta(\cdot, s)$ we denote the unique solution of the Cauchy problem

$$d\gamma(t) = \gamma(t)\, d\beta(t), \ \ \gamma(s) = 1.$$

It is known (see [9], [12]) that

$$\begin{aligned}\gamma_\beta(t,s) = \exp\big(\xi_\beta(t) - \xi_\beta(s)\big) \prod_{s<\tau\le t} \operatorname{sgn}\big(1 - d_1\beta(\tau)\big)\times \\ \times \prod_{s\le\tau<t} \operatorname{sgn}\big(1 + d_2\beta(\tau)\big) \ \text{ for } \ t > s, \\ \gamma_\beta(t,s) = \gamma_\beta^{-1}(s,t) \ \text{ for } \ t < s,\end{aligned}$$

where

$$\begin{aligned}\xi_\beta(t) &= s_0(\beta)(t) - s_0(\beta)(0) - \\ &\quad - \sum_{0<\tau\le t} \ln\big|1 - d_1\beta(\tau)\big| + \sum_{0\le\tau<t} \ln\big|1 + d_2\beta(\tau)\big| \ \text{ for } \ t > 0, \\ \xi_\beta(t) &= s_0(\beta)(t) - s_0(\beta)(0) + \\ &\quad + \sum_{t<\tau\le 0} \ln\big|1 - d_1\beta(\tau)\big| - \sum_{t\le\tau<0} \operatorname{sgn}\big|1 + d_2\beta(\tau)\big| \ \text{ for } \ t < 0.\end{aligned}$$

*Remark* 1. Let $\beta \in \mathrm{BV}([a,b],\mathbb{R})$ be such that

$$1 + (-1)^j d_j\beta(t) > 0 \ \text{ for } \ t \in [a,b] \ \ (j=1,2).$$

Let, moreover, one of the functions $\beta$, $\xi_\beta$ and $\mathcal{A}(\beta,\beta)$ be nondecreasing (nonincreasing). Then the other two functions will be nondecreasing (nonincreasing) as well.

Let $\delta > 0$. We introduce the operators

$$\nu_{1\delta}(\xi)(t) = \sup\Big\{\tau \ge t: \ \xi(\tau) \le \xi(t+) + \delta\Big\}$$

and

$$\nu_{-1\delta}(\eta)(t) = \inf\Big\{\tau \le t: \ \eta(\tau) \le \eta(t-) + \delta\Big\},$$

respectively, on the set of all nondecresing functions $\xi : \mathbb{R} \to \mathbb{R}$ and on the set of all nonincreasing functions $\eta : \mathbb{R} \to \mathbb{R}$.

$\widetilde{C}([a,b],D)$, where $D \subset \mathbb{R}^{n\times m}$, is the set of all absolutely continuous matrix-functions $X : [a,b] \to D$;

$\widetilde{C}_{loc}(\mathbb{R}_+ \setminus \{\tau_1,\tau_2,\dots\}, D)$ is the set of all matrix-functions $X : \mathbb{R}_+ \to D$ whose restriction to an arbitrary closed interval $[a,b]$ from $\mathbb{R}_+ \setminus \{\tau_1,\tau_2,\dots\}$ belongs to $\widetilde{C}([a,b],D)$.

$L([a,b],D)$ is the set of all matrix-functions $X : [a,b] \to D$, measurable and integrable.

$L_{loc}(\mathbb{R}_+ \setminus \{\tau_1, \tau_2, \dots\}, D)$ is the set of all matrix-functions $X : \mathbb{R}_+ \to D$ whose restriction to an arbitrary closed interval $[a, b]$ from $\mathbb{R}_+ \setminus \{\tau_1, \tau_2, \dots\}$ belongs to $\widetilde{C}([a, b], D)$.

If $D_1 \subset \mathbb{R}^n$ and $D_2 \subset \mathbb{R}^{n\times m}$, then $K([a, b] \times D_1, D_2)$ is the Carathéodory class, i.e., the set of all mappings $F = (f_{kj})_{k,j=1}^{n,m} : [a, b] \times D_1 \to D_2$ such that for each $i \in \{1, \dots, l\}$, $j \in \{1, \dots, m\}$ and $k \in \{1, \dots, n\}$: a) the function $f_{kj}(\cdot, x) : [a, b] \to D_2$ is measurable for every $x \in D_1$; b) the function $f_{kj}(t, \cdot) : D_1 \to D_2$ is continuous for almost all $t \in [a, b]$, and $\sup\{|f_{kj}(\cdot\,, x)| : \ x \in D_0\} \in L([a, b], \mathbb{R}; g_{ik})$ for every compact $D_0 \subset D_1$.

$K_{loc}(\mathbb{R}_+ \times D_1, D_2)$ is the set of all mappings $F : \mathbb{R}_+ \times D_1 \to D_2$ whose restriction to an arbitrary closed interval $[a, b]$ from $\mathbb{R}_+ \setminus \{\tau_1, \tau_2, \dots\}$ belongs to $K([a, b] \times D_1, D_2)$.

By a solution of the impulsive system (3), (4) we understand a continuous from the left vector-function $x \in \widetilde{C}_{loc}(\mathbb{R}_+ \setminus \{\tau_1, \tau_2, \dots\}) \cap \mathrm{BV}_{loc}\, s(\mathbb{R}_+, \mathbb{R}^n)$ satisfying both the system (1) for a.a. $t \in \mathbb{R}_+ \setminus \{\tau_k\}_{k=1}^{m_0}$ and the relation (2) for every $k \in \{1, 2, \dots\}$.

**Theorem 1.** *Let*

$$0 \le d_2(a_{i1}(t) + a_{i2}(t)) < |\eta_{ii}|^{-1} \ \textit{for} \ t \in \mathbb{R}_+ \ \ (i = 1, 2),$$

$$1 + \sigma_i d_2 a_{ii}(t) > 0 \ \textit{for} \ t \in \mathbb{R}_+ \ \ (i = 1, 2),$$

$$\sigma_i f_k(t, x_1, x_2) \operatorname{sgn} x_i \le \eta_{i1}|x_1| + \eta_{i2}|x_2| + q_k(t)$$

$$\textit{for } \mu(a_{ik})\textit{-almost all } t \in \mathbb{R}_+ \textit{ and } x_1, x_2 \in \mathbb{R} \ \ (i, k = 1, 2),$$

*and let the real part of every eigenvalue of the matrix $(\eta_{il})_{i,l=1}^2$ be negative, where $\sigma_1 = 1$, $\sigma_2 = -1$ $(\sigma_1 = \sigma_2 = -1)$, $\eta_{12}$, $\eta_{21} \in \mathbb{R}$; $\eta_{ii} < 0$ $(i = 1, 2)$, $q_k \in L_{loc}(\mathbb{R}_+, \mathbb{R}; a_{1k}) \cap L_{loc}(\mathbb{R}_+, \mathbb{R}; a_{2k})$ $(k = 1, 2)$. Let, moreover,*

$$\sigma_i \liminf_{t\to\infty} \big(\xi_{\sigma_i a_{ii}}(t) - \xi_{\sigma_i a_{ii}}(0)\big) > \delta > 0 \ \ (i = 1, 2)$$

*for some $\delta > 0$,*

$$\sup\bigg\{ \int\limits_t^{\nu_i(t)} |q_k(\tau)|\, ds_0(a_{ik})(\tau) + \sum_{t<\tau\le\nu_i(t)} (1 + \sigma_i d_2 a_{ii}(t))^{-1} |q_k(\tau)| d_2 a_{ik}(\tau) : \ t \in \mathbb{R}_+ \bigg\} < \infty \ \ (i, k = 1, 2),$$

$$s_{il} = \bigg| \int\limits_0^{+\infty} \gamma_{\sigma_i a_{ii}}(t, s)\, d\mathcal{A}(\sigma_i a_{ii}, a_{il})(s) \bigg| < \infty \ \ (i \ne l; \ \ i, l = 1, 2)$$

*and*

$$s_1 s_2 < 1,$$

*where $\nu_i(t) \equiv \nu_{\sigma_i\delta}(-\xi_{\sigma_i a_{ii}})(t)$ $(i = 1, 2)$. Then the problem* (1), (2) ((1), (3)) *is solvable.*

Consider now the impulsive system (4), (5).

Quite a number of issues of the theory of systems of differential equations with impulsive effect (both linear and nonlinear) have been studied sufficiently well (for survey of the results on impulsive systems see, e.g., [6], [8], [18]–[22], and references therein).

It is easy to show that the vector-function $x = (x_i)_{i=1}^2$ is a solution of the impulsive system (4), (5) if and only if it is a solution of the system (1), where $a_{12}(t) = a_{21}(t) \equiv 0$,

$$a_{ii}(t) \equiv t + \sum_{k:\ 0\le\tau_k<t} \alpha_{ki} \quad (i=1,2),$$

$$f_i(\tau_k, x_1, x_2) = I_{ki}(x_1, x_2) \ \text{ for } \ x_1,\ x_2 \in \mathbb{R} \ \ (i=1,2;\ \ k=1,2,\dots).$$

It is evident that $a_{ii}$ $(i=1,2)$ are nondecreasing if $\alpha_{ki} \ge 0$, $d_2 a_{ii}(\tau_k) = \alpha_{ki}$ and $d_2 a_{ii}(t) = 0$ it $t \ne \tau_k$ $(i=1,2; k=1,2,\dots)$. Moreover, they are continuous from the left. In this case

$$\xi_{\sigma_i a_{ii}} \equiv \sigma_i t + \sum_{k:\ 0\le\tau_k<t} \ln|1+\sigma_i\alpha_{ki}| \ \ (i=1,2). \tag{6}$$

**Theorem 2.** *Let*

$$0 \le \alpha_{ki} < |\eta_{ii}|^{-1} \ \ (i=1,2;\ \ k=1,2,\dots), \tag{7}$$

$$1 + \sigma_i\alpha_{ki} > 0 \ \ (i=1,2;\ \ k=1,2,\dots), \tag{8}$$

$$\sigma_i f_i(t, x_1, x_2)\operatorname{sgn} x_i \le \eta_{i1}|x_1| + \eta_{i2}|x_2| + q_i(t)$$

*for almost all* $t \in \mathbb{R}_+$ *and* $x_1,\ x_2 \in \mathbb{R}$ $(i=1,2;\ \ k=1,2,\dots)$,

$$\sigma_i I_{ki}(x_1, x_2)\operatorname{sgn} x_i \le \eta_{i1}|x_1| + \eta_{i2}|x_2| + q_{ki}$$

*for* $x_1, x_2 \in \mathbb{R}$ $(i=1,2;\ \ k=1,2,\dots)$,

*and let the real part of every eigenvalue of the matrix* $(\eta_{il})_{i,l=1}^2$ *be negative, where* $\sigma_1 = 1$, $\sigma_2 = -1$ $(\sigma_1 = \sigma_2 = -1)$, $\eta_{12}$, $\eta_{21} \in \mathbb{R}$, $\eta_{ii} < 0$ $(i=1,2)$, $q_i \in L_{loc}(\mathbb{R}_+, \mathbb{R})$ $(i=1,2)$. *Let, moreover,*

$$\liminf_{t\to\infty} \Big(t + \sigma_i \sum_{k:\ 0\le\tau_k<t} \ln(1+\sigma_i\alpha_{ki})\Big) > \delta > 0 \ \ (i=1,2) \tag{9}$$

*for some* $\delta > 0$,

$$\sup\Bigg\{ \int\limits_t^{\nu_i(t)} |q_i(\tau)|d(\tau) + \sum_{k:\ 0\le\tau_k<\nu_i(t)} (1+\sigma_i\alpha_{ki})^{-1}|q_{ki}| :\ t\in\mathbb{R}_+ \Bigg\} < \infty$$

$$(i=1,2),$$

*where the functions* $\nu_i(t) \equiv \nu_{\sigma_i\delta}(-\xi_{\sigma_i a_{ii}})(t)$ $(i=1,2)$ *are defined according to* (6). *Then the problem* (4), (5); (2) ((4), (5); (3)) *is solvable.*

*Remark* 2. By condition (7), the conditions (8) and (9) are fulfilled if $\sigma_i = 1$.

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