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ON THE INSTABILITY OF SOLUTIONS OF CERTAIN FIFTH ORDER NONLINEAR DIFFERENTIAL EQUATIONS

Abstract. The main purpose of this paper is to give sufficient conditions which guarantee the instability of the trivial solution of a nonlinear vector differential equation as follows:

$$X^{(5)} + \Psi(\dot{X}, \ddot{X})\dddot{X} + \Phi(X, \dot{X}, \ddot{X}) + \Theta(\dot{X}) + F(X) = 0.$$

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$$X^{(5)} + \Psi(\dot{X}, \ddot{X})\dddot{X} + \Phi(X, \dot{X}, \ddot{X}) + \Theta(\dot{X}) + F(X) = 0$$

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1. Introduction

It is well-known that the stability and instability behaviors of solutions of certain differential equations are very important problems in the theory and applications of differential equations. It should be noted that the true creator of the stability theory is A. M. Lyapunov [11] at the close of the 19th century. The technique discovered by him is called Lyapunov's second method or the direct method. This technique can be applied directly to the differential equation under investigation, without any knowledge of the solutions, provided the person using the method is clever enough to construct the right auxiliary functions. Up to now, many results have been obtained about the qualitative behavior of solutions of higher order nonlinear differential equations by using the method. One may refer to [13] for a survey, as well as [1-10, 14-24] and the references cited therein for some publications on the subject. However, according to our observations in the relevant literature, the results about instability of solutions of fifth order nonlinear differential equations are relatively scarce. In this direction, in the case $n = 1$, Ezeilo ([3], [4], [5]) investigated the instability of the trivial solution $x = 0$ of the following nonlinear differential equations of the fifth order:

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + a_3 \ddot{x} + a_4 \dot{x} + f(x) = 0,$$

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + h(\dot{x})\ddot{x} + g(x)\dot{x} + f(x) = 0,$$

$$x^{(5)} + \psi(\ddot{x})\dddot{x} + \phi(\ddot{x}) + \theta(\dot{x}) + f(x) = 0$$

and

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + g(\dot{x})\ddot{x} + h(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dot{x} + f(x) = 0.$$

In [21], Tiryaki also studied the instability of the trivial solution $x = 0$ of the nonlinear differential equation

$$x^{(5)} + a_1 x^{(4)} + k(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dddot{x} + g(\dot{x})\ddot{x} + h(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dot{x} + f(x) = 0.$$

Furthermore, recently, Sadek [15] discussed the subject for the fifth order nonlinear vector differential equations

$$X^{(5)} + \Psi(\ddot{X})\dddot{X} + \Phi(\ddot{X}) + \Theta(\dot{X}) + F(X) = 0$$

and

$$X^{(5)} + AX^{(4)} + B\dddot{X} + H(\dot{X})\ddot{X} + G(X)\dot{X} + F(X) = 0,$$

and Tunç [25] also gave sufficient conditions which guarantee that the trivial solution of the vector differential equations of the form

$$X^{(5)} + AX^{(4)} + \Psi(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dddot{X} +$$
$$+ G(\dot{X})\ddot{X} + H(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dot{X} + F(X) = 0$$

is unstable.

The motivation for the present study has come from the papers just mentioned above. Our aim is to acquire a similar result for a certain nonlinear vector differential equation of fifth order, which is different from those just mentioned above. Namely, in the present paper, we consider the vector differential equations of the form

$$X^{(5)} + \Psi(\dot{X}, \ddot{X})\dddot{X} + \Phi(X, \dot{X}, \ddot{X}) + \Theta(\dot{X}) + F(X) = 0 \tag{1.1}$$

in the real Euclidean space $\Re^n$ (with the usual norm denoted in what follows by $\|\cdot\|$), where $X \in \Re^n$, Ψ is a continuous $n \times n$ -symmetric matrix depending, in each case, on the arguments shown, $\Phi : \Re^n \times \Re^n \times \Re^n \to \Re^n$, $\Theta : \Re^n \to \Re^n$, $F : \Re^n \to \Re^n$ and $\Theta(0) = F(0) = 0$. It will also be supposed that the functions Φ, Θ and F are continuous.

The equation (1.1) represents a system of real fifth-order differential equations of the form

$$\begin{gathered} x_i^{(5)} + \sum_{k=1}^{n} \psi_{ik}(\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n; \ddot{x}_1, \ddot{x}_2, \ldots, \ddot{x}_n)\dddot{x}_k + \\ +\phi_i(x_1, x_2, \ldots, x_n; \dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n; \ddot{x}_1, \ddot{x}_2, \ldots, \ddot{x}_n) + \\ +\theta_i(\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n) + f_i(x_1, x_2, \ldots, x_n) = 0 \quad (i = 1, 2, \ldots, n). \end{gathered}$$

We consider through in what follows, in place of (1.1), the equivalent differential system:

$$\begin{gathered} \dot{X} = Y, \dot{Y} = Z, \quad \dot{Z} = W, \quad \dot{W} = U, \\ \dot{U} = -\Psi(Y, Z)W - \Phi(X, Y, Z)Z - \Theta(Y) - F(X) \end{gathered} \tag{1.2}$$

obtained as usual by setting $\dot{X} = Y$, $\ddot{X} = Z$, $\dddot{X} = W$, $X^{(4)} = U$ in (1.1).

The Jacobian matrices $J(\Psi(Y, Z)Z \,|Y)$, $J(\Psi(Y, Z)\,|Z)$, $J_\Theta(Y)$ and $J_F(X)$ are given by

$$\begin{gathered} J(\Psi(Y, Z)Z \,|Y) = \left(\frac{\partial}{\partial y_j} \sum_{k=1}^{n} \psi_{ik} z_k \right) = \left(\sum_{k=1}^{n} \frac{\partial \psi_{ik}}{\partial y_j} z_k \right), \\ J(\Psi(Y, Z)\,|Z) = \left(\frac{\partial}{\partial z_j} \sum_{k=1}^{n} \psi_{ik} \right) = \left(\sum_{k=1}^{n} \frac{\partial \psi_{ik}}{\partial z_j} \right), \\ J_\Theta(Y) = \left(\frac{\partial \theta_i}{\partial y_j} \right), \quad J_F(X) = \left(\frac{\partial f_i}{\partial x_j} \right), \end{gathered}$$

where $(x_1, x_2, \ldots, x_n)$, $(y_1, y_2, \ldots, y_n)$, $(z_1, z_2, \ldots, z_n)$, (ψ_{ik}), $(\theta_1, \theta_2, \ldots, \theta_n)$ and $(f_1, f_2, \ldots, f_n)$ are the components of X, Y, Z, Ψ, Θ and F, respectively. Moreover, it will also be assumed, as basic throughout what follows, that the Jacobian matrices $J(\Psi(Y, Z)\,|Y)$, $J(\Psi(Y, Z)\,|Z)$, $J_\Theta(Y)$ and $J_F(X)$ exist and are continuous and symmetric.

The symbol $\langle X, Y \rangle$ corresponding to any pair X, Y in $\Re^n$ stands for the usual scalar product $\sum_{i=1}^{n} x_i y_i$, and $\lambda_i(A)$ $(i = 1, 2, \ldots, n)$ are the eigenvalues of the $n \times n$-matrix A.

Now, we consider the linear constant coefficient fifth order differential equation

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + a_3 \ddot{x} + a_4 \dot{x} + a_5 x = 0, \tag{1.3}$$

where $a_1, a_2, \ldots, a_5$ are some real constants. It is well-known from the qualitative behavior of solutions of linear differential equations that the trivial solution of (1.3) is unstable if and only if the associated auxiliary equation

$$\psi(\lambda) \equiv \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0 \tag{1.4}$$

has at least one root with a positive real part. The existence of such a root naturally depends on (though not always all of) the coefficients $a_1, a_2, \ldots, a_5$ in (1.4). For example, if

$$a_1 < 0 \tag{1.5}$$

then it follows from a consideration of the fact that the sum of the roots of (1.4) equals to $(-a_1)$ and that at least one root of (1.4) has a positive real part for arbitrary values of $a_2, \ldots, a_5$. An analogous consideration, combined with the fact that the product of the roots (1.4) equals to (-a_5) will verify that at least one root of (1.4) has a positive real part if

$$a_1 = 0 \quad \text{and} \quad a_5 \neq 0 \tag{1.6}$$

for arbitrary a_2, a_3 and a_4. The condition $a_1 = 0$ here in (1.6) is, however, superfluous when

$$a_5 < 0, \tag{1.7}$$

for then $\psi(0) = a_5 < 0$ and $\psi(R) > 0$ if $R > 0$ is sufficiently large thus showing that there is a positive real root of (1.4) subject to (1.7) and for arbitrary a_1, a_2, a_3 and a_4.

A root with a positive real part also exists for certain equations (1.4) with a_5 positive and sufficiently large. To see this easily, we refer to the well-known Routh–Hurwitz criteria which stipulate that each root of (1.4) has a negative real part. Namely, a necessary and sufficient condition for the negativity of the real parts of all the roots of the polynomial equation (1.4) is the positivity of all the principal minors of the Hurwitz matrix

$$H_5 = \begin{bmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & 0 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & 0 & a_5 \end{bmatrix}.$$

It should be noted that the principal diagonal of the Hurwitz matrix H_5 exhibits the coefficients of the polynomial equation (1.4) in the order of their numbers from a_1 to a_5. The fourth order minor, say Δ_4, concerned here is given by the determinant

$$\Delta_4 = \begin{vmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ 0 & 0 & a_5 & a_4 \end{vmatrix},$$

that is, on multiplying out,

$$\Delta_4 = -a_5^2 + a_5(2a_1a_4 + a_2a_3 - a_1a_2^2) + a_4(a_1a_2a_3 - a_3^2 - a_1^2a_4). \qquad (1.8)$$

It is thus clear, in particular, that if $\Delta_4 < 0$, as would indeed be the case from (1.8) if

$$a_5 \geq R_0 > 0 \qquad (1.9)$$

with $R_0 = R_0(a_1, a_2, a_3, a_4)$ sufficiently large, then at least one root of (1.4) has a non-negative real part subject to (1.9).

2. Main Result

We establish the following

Theorem 2.1. *In addition to the basic assumptions imposed on Ψ, Φ, Θ and F, suppose that the following conditions are satisfied:*

(i) *The matrices $J_\Theta(Y)$, $J_F(X)$ are symmetric and $\lambda_i(J_F(X)) < 0$ for all $X \in \Re^n$ $(i = 1, 2, \ldots, n)$;*

(ii) $\sum_{i=1}^{n} z_i\phi_i(X, Y, Z) \geq 0$ *for all X, Y,$Z \in \Re^n$, where* $\Phi(X, Y, Z) = (\phi_1(X, Y, Z), \ldots, \phi_n(X, Y, Z))$;

(iii) *The matrices $\Psi(Y, Z)$ and $J(\Psi(Y, Z)Z\,|Y)$ are symmetric and $J(\Psi(Y, Z)Z\,|Y)$ is negative-definite for all Y,$Z \in \Re^n$,*

or

(i)′ *The matrices $J_\Theta(Y)$, $J_F(X)$ are symmetric and $\lambda_i(J_F(X)) > 0$ for all $X \in \Re^n$ $(i = 1, 2, \ldots, n)$;*

(ii)′ $\sum_{i=1}^{n} z_i\phi_i(X, Y, Z) \leq 0$ *for all X, Y, $Z \in \Re^n$, where* $\Phi(X, Y, Z) = (\phi_1(X, Y, Z), \ldots, \phi_n(X, Y, Z))$;

(iii)′ *The matrices $\Psi(Y, Z)$ and $J(\Psi(Y, Z)Z\,|Y)$ are symmetric and $J(\Psi(Y, Z)Z\,|Y)$ is positive-definite for all Y,$Z \in \Re^n$.*

Then the trivial solution $X = 0$ of (1.2) *is unstable.*

Remark 2.2. It should be noted that, for the case $n = 1$, the result of Ezeilo [3; Theorem 3] is a special case of our result. The result established here includes and improves the result established by Sadek [15; Theorem 3]. The following lemma is important for the proof of the theorem.

Lemma 2.3. *Let A be a real symmetric $n \times n$-matrix and*

$$a' \geq \lambda_i(A) \geq a > 0 \quad (i = 1, 2, \ldots, n),$$

where a', a are constants. Then

$$a' \langle X, X\rangle \geq \langle AX, X\rangle \geq a \langle X, X\rangle$$

and

$$a'^2 \langle X, X\rangle \geq \langle AX, AX\rangle \geq a^2 \langle X, X\rangle .$$

Proof. See [12]. □

Proof of the theorem. The main tool in the proof of the theorem is the Lyapunov function $V_0 = V_0(X, Y, Z, W, U)$ defined as follows:

$$V_0 = \frac{1}{2}\langle W, W\rangle - \langle Y, F(X)\rangle - \langle Z, U\rangle - \int_0^1 \langle \Theta(\sigma Y), Y\rangle \, d\sigma - \\ - \int_0^1 \langle \sigma\Psi(Y, \sigma Z)Z, Z\rangle \, d\sigma. \tag{2.1}$$

Clearly, it follows from (2.1) that $V_0(0, 0, 0, 0, 0) = 0$. Obviously, it also follows from the assumptions of the theorem, the above lemma and (2.1) that

$$V_0(0, 0, 0, \varepsilon, 0) = \frac{1}{2}\langle \varepsilon, \varepsilon\rangle = \frac{1}{2}\|\varepsilon\|^2 > 0$$

for all arbitrary $\varepsilon \neq 0$, $\varepsilon \in \Re^n$. Thus, in every neighborhood of $(0, 0, 0, 0, 0)$ there exists a point $(\xi, \eta, \zeta, \mu, \tau)$ such that $V_0(\xi, \eta, \zeta, \mu, \tau) > 0$ for all ξ, η, ζ, μ, τ in $\Re^n$. Next, let $(X, Y, Z, W, U) = (X(t), Y(t), Z(t), W(t), U(t))$ be an arbitrary solution of the system (1.2). Then from (2.1) and (1.2) we have by an elementary differentiation that

$$\dot{V}_0 = \frac{d}{dt}V_0(X, Y, Z, W, U) = \langle Z, \Phi(X, Y, Z)\rangle - \langle Y, J_F(X)Y\rangle + \\ + \langle \Psi(Y, Z)W, Z\rangle + \langle \Theta(Y), Z\rangle - \\ - \frac{d}{dt}\int_0^1 \langle \Theta(\sigma Y), Y\rangle \, d\sigma - \frac{d}{dt}\int_0^1 \langle \sigma\Psi(Y, \sigma Z)Z, Z\rangle \, d\sigma. \tag{2.2}$$

But

$$\frac{d}{dt}\int_0^1 \langle \sigma\Psi(Y, \sigma Z)Z, Z\rangle \, d\sigma = \int_0^1 \langle \sigma\Psi(Y, \sigma Z)Z, W\rangle \, d\sigma + \int_0^1 \langle \sigma\Psi(Y, \sigma Z)W, Z\rangle \, d\sigma + \\ + \int_0^1 \left\langle \sigma^2 J(\Psi(Y, \sigma Z)\,|Z)WZ \;, Z\right\rangle d\sigma + \int_0^1 \langle \sigma J(\Psi(Y, \sigma Z)Z\,|Y)Z \;, Z\rangle \, d\sigma = \\ = \int_0^1 \langle \sigma\Psi(Y, \sigma Z)W, Z\rangle \, d\sigma + \int_0^1 \sigma \frac{\partial}{\partial\sigma}\langle \sigma\Psi(Y, \sigma Z)W, Z\rangle \, d\sigma +$$

$$+\int_0^1 \langle \sigma J(\Psi(Y,\sigma Z)Z\,|Y)Z\ ,Z\rangle\, d\sigma =$$

$$= \sigma^2 \langle \Psi(Y,\sigma Z)W,Z)\rangle \big|_0^1 + \int_0^1 \langle \sigma J(\Psi(Y,\sigma Z)Z\,|Y)Z\ ,Z\rangle\, d\sigma =$$

$$= \langle \Psi(Y,Z)W,Z\rangle + \int_0^1 \langle \sigma J(\Psi(Y,\sigma Z)Z\,|Y)Z\ ,Z\rangle\, d\sigma \tag{2.3}$$

and

$$\frac{d}{dt}\int_0^1 \langle \Theta(\sigma Y),Y\rangle\, d\sigma = \int_0^1 \sigma \langle J_\Theta(\sigma Y)Z,Y\rangle d\sigma + \int_0^1 \langle \Theta(\sigma Y),Z\rangle\, d\sigma =$$

$$= \int_0^1 \sigma \frac{\partial}{\partial\sigma}\langle \Theta(\sigma Y),Z\rangle\, d\sigma + \int_0^1 \langle \Theta(\sigma Y),Z\rangle\, d\sigma =$$

$$= \sigma \langle \Theta(\sigma Y),Z\rangle \big|_0^1 = \langle \Theta(Y),Z\rangle\ . \tag{2.4}$$

Substituting the estimates (2.3) and (2.4) into (2.2), we obtain

$$\dot V_0 = \langle Z, \Phi(X,Y,Z)\rangle - \langle Y, J_F(X)Y\rangle - \int_0^1 \langle \sigma J(\Psi(Y,\sigma Z)Z\,|Y)Z\ ,Z\rangle\, d\sigma.$$

Hence, the assumptions **(i)**, **(ii)** and **(iii)** of the theorem and the lemma show that $\dot V_0(t) \geq 0$ for all $t \geq 0$, that is, $\dot V_0$ is positive semi-definite. Furthermore, $\dot V_0 = 0$ $(t \geq 0)$ necessarily implies (only) that $Y = 0$ for all $t \geq 0$, and therefore also that $X = \xi$ (a constant vector), $Z = \dot Y = 0$, $W = \ddot Y = 0$, $U = \dddot Y = 0$, for all $t \geq 0$. Substituting the estimates

$$X = \xi, \quad Y = Z = W = U = 0$$

into (1.2), we obtain that $F(\xi) = 0$ which necessarily implies that $\xi = 0$ because of $F(0) = 0$. Hence

$$X = Y = Z = W = U = 0 \quad \text{for all} \quad t \geq 0.$$

Therefore, the function V_0 satisfies all the conditions of the Krasovskiĭ criterion [8] if the conditions of the theorem hold. Thus, the basic properties of the function $V_0(X,Y,Z,W,U)$, which are proved just above verify that the zero solution of the system (1.2) is unstable. (See Theorem 1.15 in Reissig [13] and Krasovskiĭ [8]). The system of equations (1.2) is equivalent to the differential equation (1.1). Consequently, the original statement of the first part of the theorem follows.

Similarly, for the proof of the second part of the theorem, we consider the Lyapunov function $V_1 = V_1(X,Y,Z,W,U)$ defined as $V_1 = -V_0$, where V_0 is defined by (2.1). The remaining proof of the second part of the theorem follows the lines indicated in the proof of the first part just shown above, except for some minor modifications. We will omit the details. (See also the result of Ezeilo [3; Theorem 3]).

Example 2.4. As a special case of (1.1) (see Sadek [15]), if we take for $n = 3$

$$\Psi = \begin{bmatrix} z_1 & 1 & 2 \\ 1 & z_2 & 3 \\ 2 & 3 & z_3 \end{bmatrix}, \quad \Phi = \begin{bmatrix} z_1^3 + z_1^5 \\ z_2^3 + z_2^5 \\ z_3^3 + z_3^5 \end{bmatrix},$$

$$\Theta = \begin{bmatrix} y_1^2 \\ y_2^2 \\ y_3^2 \end{bmatrix}, \quad F = \begin{bmatrix} -x_1 - x_1^3 \\ -x_2 - x_2^3 \\ -x_3 - x_3^3 \end{bmatrix},$$

then we will have

$$J_\Theta(Y) = \begin{bmatrix} 2y_1 & 0 & 0 \\ 0 & 2y_2 & 0 \\ 0 & 0 & 2y_3 \end{bmatrix},$$

$$J_F(X) = \begin{bmatrix} -1 - 3x_1^2 & 0 & 0 \\ 0 & -1 - 3x_2^2 & 0 \\ 0 & 0 & -1 - 3x_3^2 \end{bmatrix}$$

and

$$\lambda_1(J_F) = -1 - 3x_1^2, \lambda_2(J_F) = -1 - 3x_2^2, \lambda_3(J_F) = -1 - 3x_3^2.$$

Hence, $J_F(X) < 0$ for all x_1, x_2, x_3, and

$$\sum_{i=1}^{3} z_i \Phi_i(Z) = z_1^4 + z_1^6 + z_2^4 + z_2^6 + z_3^4 + z_3^6 \text{ for all } z_1, z_2, z_3.$$

Thus all the conditions of the first part of the theorem are satisfied.

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