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TO THE QUESTION OF OSCILLATION OF SOLUTIONS OF TWO-DIMENSIONAL DIFFERENTIAL SYSTEMS WITH DEVIATED ARGUMENTS

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Consider the system of differential inequalities

$$\begin{aligned} &u_1'(t)\operatorname{sign} u_2(\sigma(t)) \geq p(t)|u_2(\sigma(t))|, \\ &u_2'(t)\operatorname{sign} u_1(\tau(t)) \leq -q(t)|u_1(\tau(t))|, \end{aligned} \tag{1}$$

where $p, q : \mathbb{R}_+ \to \mathbb{R}_+$ are locally summable functions, $\tau, \sigma : \mathbb{R}_+ \to \mathbb{R}$ are nondecreasing continuous functions and $\sigma(t) \leq t$, $\sigma(\tau(t)) \leq t$ for $t \in \mathbb{R}_+$, $\lim_{t\to+\infty} \tau(t) = +\infty$, $\lim_{t\to+\infty} \sigma(t) = +\infty$.

Definition. A *proper* solution (u_1, u_2) of the system (1), i.e. a nontrivial solution defined in some neighbourhood of $+\infty$, is said to be *oscillatory* if both u_1 and u_2 have sequences of zeros tending to infinity. If there exists $t_0 \in \mathbb{R}_+$ such that $u_1(t)u_2(t) \neq 0$ for $t \geq t_0$, then (u_1, u_2) is said to be *nonoscillatory.*

In this paper, we are especially interested in the question whether every proper solution of (1) is oscillatory.

In the sequel, we assume that the condition

$$\int_0^{+\infty} p(s)ds = +\infty$$

is fulfilled.

Theorem 1. *Let for some $k \in \mathbb{N}$ and for any $t_0 \in \mathbb{R}_+$ the inequality*

$$\limsup_{t\to+\infty} \bigg(\int_{\sigma(\tau(t))}^{t} q(s)\omega_k\Big(\sigma(\tau(s)), \sigma(\tau(t)); t_0\Big) \int_{\eta_k(t_0)}^{\tau(s)} p(\xi)d\xi ds + \\ + g_k\Big(\sigma(\tau(t)); t_0\Big) \int_t^{+\infty} q(s)ds \bigg) > 1,$$

be fulfilled, where

$$g_k(t; t_0) = \int_{\eta_k(t_0)}^{t} p(s)\omega_k\Big(\sigma(s), s; t_0\Big)ds +$$

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$$+\int_{\eta_k(t_0)}^{t} q(s)\psi_k(s;t_0)\int_{\eta_k(t_0)}^{s} p(\xi)\omega_k\Big(\sigma(\xi),\xi;t_0\Big)d\xi ds \quad for \quad t\geq \eta_k(t_0),$$

$$\omega_j(t,s;t_0)=\exp\bigg\{\int_{t}^{s} q(\xi)\varphi_j(\xi;t_0)\int_{\eta_j(t_0)}^{\tau(\xi)} p(\xi_1)d\xi_1 d\xi\bigg\}$$

$$for \quad s\geq t\geq \eta_j(t_0) \quad (j=1,\dots,k),$$

$$\varphi_1(t;t_0)=1, \quad \varphi_j(t;t_0)=\omega_{j-1}\Big(\sigma(\tau(t)),t;t_0\Big) \quad for \quad t\geq \eta_j(t_0) \quad (j=2,\dots,k),$$

$$\psi_1(t;t_0)=0, \quad \psi_j(t;t_0)=\int_{\eta_{j-1}(t_0)}^{\sigma(\tau(t))} p(s)\exp\bigg\{\int_{\sigma(s)}^{t} q(\xi)\psi_{j-1}(\xi;t_0)d\xi\bigg\}ds \tag{2}$$

$$for \quad t\geq \eta_j(t_0) \quad (j=2,\dots,k),$$

$$\eta_1(t)=\max\Big\{s: \min\Big(\sigma(\tau(\sigma(s))),\sigma(s)\Big)\leq t\Big\}, \quad \eta_j(t)=\eta_1\Big(\eta_{j-1}(t)\Big)$$

$$(j=2,\dots,k).$$

Then every proper solution of (1) *is oscillatory.*

(In the case of second order differential inequalities, an analogous construction of the functions φ_k, ψ_k, η_k can be found in [1]).

Theorem 2. *Let for some* $k\in\mathbb{N}$ *and for any* $t_0\in\mathbb{R}_+$ *the inequality*

$$\limsup_{t\to+\infty}\psi_k(t;t_0)\int_{t}^{+\infty} q(s)ds>1,$$

be fulfilled, where the function ψ_k *is defined by* (2).

Then every proper solution of (1) *is oscillatory.*

References

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