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ON OSCILLATION OF SECOND ORDER LINEAR DIFFERENCE EQUATIONS WITH DEVIATED ARGUMENTS

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Consider the equation

$$\Delta^2 u(k) + \sum_{j=1}^{m} p_j(k) u\big(\tau_j(k)\big) = 0 \quad k = 1, 2, \dots, \tag{1}$$

where $\Delta u(k) = u(k+1) - u(k)$, $p_j : N \to R_+$, $\tau_j : N \to N$, and $\lim\limits_{k\to+\infty} \tau_j(k) = +\infty$ $(j = 1, \dots, m)$. A sequence $\{u(k)\}_{k=1}^{+\infty}$ is said to be a proper solution of the equation (1) if it satisfies (1) for any $k = 1, 2, \dots$ and

$$\sup\Big\{|u(i)| : i \geq k\Big\} > 0 \quad \text{for} \quad k \in N.$$

A solution $u(k)$ of the equation (1) is said to be nonoscillatory if there exists $k_0 \in N$ such that either $u(k) > 0$ or $u(k) < 0$ for $k \geq k_0$. Otherwise the solution is called oscillatory.

Below sufficient conditions are given for all proper solutions of (1) to be oscillatory as well as for a nonoscillatory solution to exist.

Theorem 1. *Suppose that*

$$\varliminf_{k\to+\infty} \frac{\tau_j(k)}{k} > 0 \quad (j = 1, \dots, m) \tag{2}$$

and for any $\lambda \in [0, 1[$ *there exists* $\varepsilon > 0$ *such that*

$$\varliminf_{k\to+\infty} k^{1-\lambda} \sum_{i=k}^{+\infty} \sum_{j=1}^{m} p_j(i) \big(\tau_j(i)\big)^{\lambda} > \lambda + \varepsilon.$$

Then every proper solution of (1) *is oscillatory.*

Theorem 2. *Suppose that* (2) *is fulfilled and for any* $\lambda \in [0, 1[$ *there exists* $\varepsilon > 0$ *such that*

$$\varliminf_{k\to+\infty} k \sum_{i=k}^{+\infty} \sum_{j=1}^{m} p_j(i) \Big(\frac{\tau_j(i)}{i}\Big)^{\lambda} > \lambda(1-\lambda) + \varepsilon.$$

Then every proper solution of (1) *is oscillatory.*

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Theorem 3. *Suppose that*

$$\varliminf_{k\to+\infty} k\sum_{i=k}^{+\infty} p(i) > \max\left\{\lambda(1-\lambda)\left(\sum_{j=1}^{m} c_j\alpha_j\right)^{-\lambda} : \lambda\in[0,1]\right\},$$

where $p : N \to R_+$, $0 < \alpha_j = \varliminf_{k\to+\infty} \dfrac{\tau_j(k)}{k}$, $c_j \in]0,+\infty[$ $(j = 1,\dots,m)$. *Then every proper solution of the equation*

$$\Delta^2 u(k) + p(k)\sum_{j=1}^{m} c_j u\big(\tau_j(k)\big) = 0$$

is oscillatory.

Corollary. *Suppose that*

$$\varliminf_{k\to+\infty} k\sum_{i=k}^{+\infty} p(i) > \frac{1}{4},$$

where $p : N \to R_+$. *Then every solution of the equation*

$$\Delta^2 u(k) + p(k)u(k) = 0$$

is oscillatory.

Theorem 4. *Suppose that for some* $\lambda \in]0,1[$ *there exists* $k_0 \in N$ *such that*

$$k^{1-\lambda}\sum_{i=k}^{+\infty} p(i)\big(\tau(i)\big)^{\lambda} \le \lambda \quad \text{for} \quad k \ge k_0.$$

Then (1) *has a nonoscillatory solution.*

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